

Communication in a Poisson Field of Interferers—Part I: Interference Distribution and Error Probability

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Abstract—We present a mathematical model for communication subject to both network interference and noise. We introduce a framework where the interferers are scattered according to a spatial Poisson process, and are operating asynchronously in a wireless environment subject to path loss, shadowing, and multipath fading. We consider both cases of slow and fast-varying interferer positions. The paper is comprised of two separate parts. In Part I, we determine the distribution of the aggregate network interference at the output of a linear receiver. We characterize the error performance of the link, in terms of average and outage probabilities. The proposed model is valid for any linear modulation scheme (e.g., M -ary phase shift keying or M -ary quadrature amplitude modulation), and captures all the essential physical parameters that affect network interference. Our work generalizes the conventional analysis of communication in the presence of additive white Gaussian noise and fast fading, allowing such results to account for the effect of network interference. In Part II of the paper, we derive the capacity of the link when subject to network interference and noise, and characterize the spectrum of the aggregate interference.

Index Terms—Stochastic geometry, Poisson field, aggregate network interference, error probability, stable laws.

I. INTRODUCTION

IN a wireless network composed of many spatially scattered nodes, there are several fundamental impairments that constrain the communication between nodes, including *thermal noise* and *network interference*. Thermal noise is introduced by the receiver electronics and is usually modeled as additive white Gaussian noise (AWGN), which constitutes a good approximation in most cases. Interference, on the other hand, is due to signals radiated by other transmitters, which undesirably affect receiver nodes in the same or in a different network. For simplicity, interference is typically approximated by AWGN with some given power [1]. However,

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this elementary model does not completely capture the physical parameters that affect interference, namely: 1) the spatial distribution of nodes scattered in the network; 2) the transmission characteristics of nodes, such as modulation, power, and synchronization; and 3) the propagation characteristics of the medium, such as path loss, shadowing, and multipath fading. If a spatial Poisson process is used to model the user positions, then all these parameters can be easily accounted for, and appear explicitly in the resulting performance expressions.

The application of the Poisson field model to cellular networks was investigated in [2] and later advanced in [3]. However, these papers either ignore random propagation effects (such as shadowing and multipath fading), or restrict the analysis to error probability in non-coherent FSK modulations. In other related work [4], it is assumed that the different interferers are synchronized at the symbol or slot level, which may be unrealistic in many situations. In [5], the authors choose a different approach and restrict the node locations to a disk or a ring in the two-dimensional plane. Although this ensures that the number of interferers is finite, it complicates the analysis and provides limited insights into the effect of network interference. In [6]–[8], the authors analyze coexistence issues in wireless networks, but consider only a small, fixed number of interferers. Lastly, none of the mentioned studies attempts a system characterization that simultaneously incorporates metrics such as error probability, channel capacity, and power spectral density.

In this two-part paper, we introduce a framework where the interferers are scattered according to a spatial Poisson process, and are operating asynchronously in a wireless environment subject to path loss, shadowing, and multipath fading. We accomplish such goal by using fundamental tools from stochastic geometry.¹ In Part I of the paper, we determine the statistical distribution of the aggregate network interference at the output of a linear receiver, located anywhere in the two-dimensional plane. We provide expressions for the error performance of the link (in terms of average and outage probabilities), which are valid for any linear modulation scheme. We then quantify these metrics as a function of various important system parameters, such as the signal-to-noise ratio (SNR), interference-to-noise ratio (INR), path loss exponent, and spatial density of the interferers. Our analysis clearly shows how the system performance depends on these parameters, thereby providing insights that

¹An overview on the application of stochastic geometry to various problems in wireless communication can be found in the tutorial papers [9], [10].

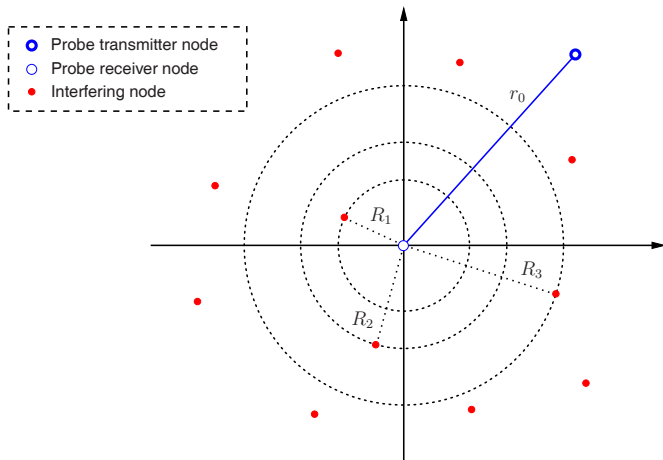


Fig. 1. Poisson field model for the spatial distribution of nodes. Without loss of generality, we assume the origin of the coordinate system coincides with the probe receiver.

may be of value to the network designer. In Part II of the paper [11], we derive the capacity of the link when subject to network interference and noise, and characterize the spectrum of the aggregate interference.

This paper is organized as follows. Section II describes the system model. Section III derives the representation and distribution of the aggregate interference. Section IV analyzes the error performance of the system, and illustrates its dependence on important network parameters. Section V concludes the paper and summarizes important findings.

II. SYSTEM MODEL

A. Spatial Distribution of the Nodes

We model the spatial distribution of the nodes according to a homogeneous Poisson point process Π in the two-dimensional infinite plane. The Poisson point process [12] has been successfully used in the context of wireless networks, most notably in what concerns connectivity and coverage [13]–[15], throughput [16]–[18], physical-layer security [19], and coexistence [20], among other topics. Typically, the terminal positions are unknown to the network designer a priori, so we may as well treat them as completely random according to a spatial Poisson process.²

The Poisson point process Π has a single parameter λ that corresponds to the spatial density of interfering nodes, in nodes per unit area. We define the *interfering nodes* to be the set of terminals which are transmitting within the frequency band of interest, during the time interval of interest, and hence are effectively contributing to the total interference. Then, irrespective of the network topology (e.g., point-to-point or broadcast) or multiple-access technique (e.g., time or frequency hopping), the above model depends only on the density λ of interfering nodes.³ The proposed spatial model is

²The spatial Poisson process is a natural choice in such situation because, given that a node is inside a region \mathcal{R} , the probability density function (p.d.f.) of its position is conditionally uniform over \mathcal{R} .

³Time and frequency hopping can be easily accommodated in this model, using the splitting property of Poisson processes [21] to obtain the *effective* density of nodes that contribute to the interference.

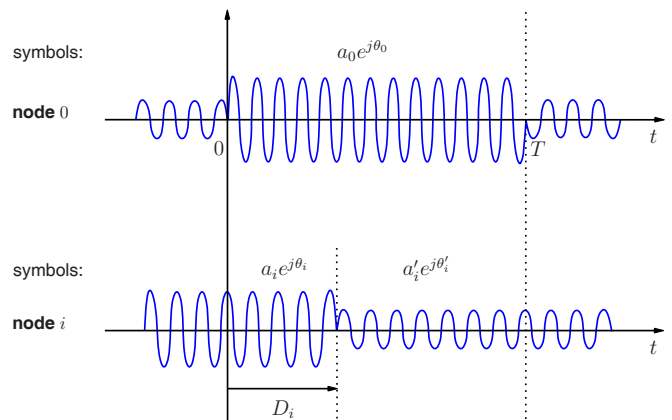


Fig. 2. Asynchronism between different transmitting nodes. In the observation interval $[0, T]$, a change in constellation symbol of node i occurs at random time $t = D_i$, from $a_i e^{j\theta_i}$ to $a'_i e^{j\theta'_i}$, where a and θ denote the transmitted symbol amplitude and phase, respectively. The distribution of D_i is assumed to be $\mathcal{U}(0, T)$. Therefore, node 0 initiates symbol transmissions at times nT by convention, while node i initiates symbol transmissions at times $nT + D_i$.

depicted in Fig. 1. For analytical purposes, we assume there is a *probe link* composed of two nodes: the *probe receiver*, located at the origin of the two-dimensional plane (without loss of generality), and the *probe transmitter* (node $i = 0$), deterministically located at a distance r_0 from the origin. All the other nodes ($i = 1 \dots \infty$) are interfering nodes, whose random distances to the origin are denoted by $\{R_i\}_{i=1}^{\infty}$, where $R_1 \leq R_2 \leq \dots$. Our goal is then to determine the effect of the interfering nodes on the probe link.

B. Transmission Characteristics of the Nodes

Concerning the transmission characteristics of users, we consider that all interfering nodes employ the same linear modulation scheme, such as M -ary phase shift keying (M -PSK) or M -ary quadrature amplitude modulation (M -QAM), with symbol period T . Furthermore, they all transmit at the same power P – a plausible constraint when power control is too complex to implement (e.g., decentralized ad-hoc networks). For generality, however, we allow the probe transmitter to employ an arbitrary linear modulation and arbitrary power P_0 , not necessarily equal to those used by the interfering nodes.

The assumption of synchronization among interfering nodes is not necessary in our analysis. Instead, we consider asynchronous transmissions where different terminals are allowed to operate independently. As depicted in Fig. 2, node i transmits with a random delay D_i relative to node 0, where $D_i \sim \mathcal{U}(0, T)$.⁴ We consider that the probe receiver employs a conventional coherent demodulator, capable of estimating the shadowing and fading affecting its own link, and hence ensuring that coherent demodulation of the desired signal is possible. Typically, parameters such as the spatial density of interferers and the propagation characteristics of the medium (e.g., shadowing and path loss parameters) are unknown to the receiver. This lack of information about the interference, together with constraints on receiver complexity, justify the

⁴We use $\mathcal{U}(a, b)$ to denote a real uniform distribution in the interval $[a, b]$.

use of a simple linear demodulator, which is optimal when only AWGN is present.

C. Propagation Characteristics of the Medium

To account for the propagation characteristics of the environment, we consider that the median of the signal amplitude decays with the distance r according to k/r^b , for some given constant k . The amplitude loss exponent b is environment-dependent, and can approximately range from 0.8 (e.g., hallways inside buildings) to 4 (e.g., dense urban environments), where $b = 1$ corresponds to free space propagation [22].⁵ The use of such a decay law also ensures that interferers located far away have a negligible contribution to the total interference observed at the probe receiver, thus making the infinite-plane model reasonable.

To capture the shadowing effect, we model the channel amplitude gain S as a log-normal random variable (r.v.) such that $S = \mu e^{\sigma G}$, where $G \sim \mathcal{N}(0, 1)$,⁶ $\mu = k/r^b$ is the median of S , and σ is the shadowing coefficient.⁷ Thus, the shadowing is responsible for random fluctuations of the channel gain around the median path gain k/r^b . The multipath effect is modeled as fast fading, which is superimposed on the path loss and shadowing. Specifically, the fading affects the received signal by introducing a random phase $\phi \sim \mathcal{U}(0, 2\pi)$, as well as an amplitude factor α with arbitrary distribution and normalized to have unit power gain, i.e., $\mathbb{E}\{\alpha^2\} = 1$.⁸ Because of its fast nature, the fading due to local scattering is always averaged out in this paper, both when determining the interference distribution and the error probability. In what follows, we consider the shadowing (and similarly for the fading) to be independent for different nodes i , and approximately constant during at least one symbol interval.

D. Mobility and Session Lifetime of the Interferers

Typically, the time variation of the distances $\{R_i\}_{i=1}^{\infty}$ of the interferers is highly coupled with that of the shadowing $\{G_i\}_{i=1}^{\infty}$ affecting those nodes. This is because the shadowing is itself associated with the movement of the nodes near large blocking objects. Thus, we introduce the notation \mathcal{P} to denote “a particular realization of the distances $\{R_i\}_{i=1}^{\infty}$ and shadowing $\{G_i\}_{i=1}^{\infty}$ of the interferers,” or more succinctly, “the position of the interferers.” In this paper, we analyze the following two scenarios, which differ in the speed of variation of \mathcal{P} :

⁵Note that the amplitude loss exponent is b , while the corresponding power loss exponent is $2b$.

⁶We use $\mathcal{N}(\mu, \sigma^2)$ to denote a real Gaussian distribution with mean μ and variance σ^2 .

⁷This model for combined path loss and log-normal shadowing can be expressed in logarithmic form [22], [23], such that the channel loss in dB is given by $L_{\text{dB}} = k_0 + k_1 \log_{10} r + \sigma_{\text{dB}} G$, where $G \sim \mathcal{N}(0, 1)$. The environment-dependent parameters $(k_0, k_1, \sigma_{\text{dB}})$ can be related to (k, b, σ) as follows: $k_0 = -20 \log_{10} k$, $k_1 = 20b$, and $\sigma_{\text{dB}} = \frac{20}{\ln 10} \sigma$. The parameter σ_{dB} is the standard deviation of the channel loss in dB (or, equivalently, of the received SNR in dB), and typically ranges from 6 to 12.

⁸We use $\mathbb{E}\{\cdot\}$ and $\mathbb{V}\{\cdot\}$ to denote the expectation and variance operators, respectively.

- 1) *Slow-varying \mathcal{P}* : During the interval of interest (e.g., a symbol or packet time), the distance R_i of each interferer is approximately constant, $R_i(t) \approx R_i$. Furthermore, the interferers have a long session lifetime, transmitting continuously over many symbols. In this quasi-static scenario, \mathcal{P} varies slowly with time, and thus it is insightful to condition the interference analysis on a given realization of \mathcal{P} . As we shall see, this naturally leads to the derivation of the *error outage probability* of the probe link, which in this case is a more meaningful metric than the error probability averaged over \mathcal{P} [24]–[26].
- 2) *Fast-varying \mathcal{P}* : As in the previous case, $R_i(t) \approx R_i$ during the interval of interest. However, the interferers have a short session lifetime, where each node periodically becomes active, transmits a burst of symbols, and then turns off (e.g., in sensor or packet networks). Then, the set of *interfering nodes* (the set of nodes that are transmitting and contributing to the interference) changes often, and so does their effective position \mathcal{P} , which experiences a variation analogous to that of a block fading model. In this dynamic scenario, it is insightful to average the interference analysis over all possible realizations of \mathcal{P} , which naturally leads to the derivation of the *average error probability* of the probe link.

III. INTERFERENCE REPRESENTATION AND DISTRIBUTION

A. Complex Baseband Representation of the Interference

Under the system model described in Section II, the aggregate signal $Z(t)$ at the probe receiver can be written for $0 \leq t \leq T$ as

$$Z(t) = \frac{k\alpha_0 e^{\sigma G_0}}{r_0^b} \sqrt{\frac{2}{T}} a_0 \cos(2\pi f_c t + \theta_0) + Y(t) + W(t),$$

where the first right-hand term is the desired signal from the transmitter probe node, $Y(t)$ is the aggregate interference given in (1) at the top of the next page, and $W(t)$ is the AWGN with two-sided power spectral density $N_0/2$, independent of $Y(t)$. In the above equations, we use the following notation: T is the symbol period; f_c is the carrier frequency; $a_i e^{j\theta_i}$ and $a'_i e^{j\theta'_i}$ are r.v.'s denoting successive constellation symbols transmitted by the node i during the interval of interest $[0, T]$ (see Fig. 2); and $u(t)$ is the unit step function. The overall effect of the path loss, log-normal shadowing, and fading on node i is captured by the amplitude factor $k\alpha_i e^{\sigma G_i} / R_i^b$, where $G_i \sim \mathcal{N}(0, 1)$, and by the uniform phase ϕ_i .⁹ We consider that the r.v.'s α_i , ϕ_i , G_i , R_i , $a_i e^{j\theta_i}$, $a'_i e^{j\theta'_i}$, and D_i are mutually independent for a given node i , and that the sequences $\{\alpha_i\}$, $\{\phi_i\}$, $\{G_i\}$, $\{a_i e^{j\theta_i}\}$, $\{a'_i e^{j\theta'_i}\}$, and $\{D_i\}$ are independent identically distributed (i.i.d.) in i .

The probe receiver demodulates the desired signal from the aggregate signal $Z(t)$, using a conventional linear demodulator. This can be achieved by projecting $Z(t)$ onto the orthonormal set $\{\psi_1(t) = \sqrt{2/T} \cos(2\pi f_c t),$

⁹Since we assume that the probe receiver can perfectly estimate the phase ϕ_0 associated with the multipath fading affecting its own link, we set $\phi_0 = 0$ without loss of generality.

$$Y(t) = \sum_{i=1}^{\infty} \frac{k\alpha_i e^{\sigma G_i}}{R_i^b} \left(\sqrt{\frac{2}{T}} a_i \cos(2\pi f_c t + \theta_i + \phi_i) u(D_i - t) + \sqrt{\frac{2}{T}} a'_i \cos(2\pi f_c t + \theta'_i + \phi_i) u(t - D_i) \right), \quad 0 \leq t \leq T \quad (1)$$

$\psi_2(t) = -\sqrt{2/T} \sin(2\pi f_c t)$ }. Defining the in-phase and quadrature (IQ) components $Z_n = \int_0^T Z(t) \psi_n(t) dt$, $n = 1, 2$, we can write

$$Z_1 = \frac{k\alpha_0 e^{\sigma G_0}}{r_0^b} a_0 \cos \theta_0 + Y_1 + W_1 \quad (2)$$

$$Z_2 = \frac{k\alpha_0 e^{\sigma G_0}}{r_0^b} a_0 \sin \theta_0 + Y_2 + W_2, \quad (3)$$

where W_1 and W_2 are $\mathcal{N}(0, N_0/2)$ and mutually independent. After some algebra (Appendix A), Y_1 and Y_2 can be expressed as

$$Y_n = \int_0^T Y(t) \psi_n(t) dt = \sum_{i=1}^{\infty} \frac{e^{\sigma G_i} X_{i,n}}{R_i^b}, \quad n = 1, 2, \quad (4)$$

where

$$X_{i,1} = k\alpha_i \left[a_i \frac{D_i}{T} \cos(\theta_i + \phi_i) + a'_i \left(1 - \frac{D_i}{T} \right) \cos(\theta'_i + \phi_i) \right] \quad (5)$$

$$X_{i,2} = k\alpha_i \left[a_i \frac{D_i}{T} \sin(\theta_i + \phi_i) + a'_i \left(1 - \frac{D_i}{T} \right) \sin(\theta'_i + \phi_i) \right]. \quad (6)$$

Using complex baseband notation,¹⁰ equations (2)-(6) can be further simplified as

$$\mathbf{Z} = \frac{k\alpha_0 e^{\sigma G_0}}{r_0^b} a_0 e^{j\theta_0} + \mathbf{Y} + \mathbf{W} \quad (7)$$

$$\mathbf{Y} = \sum_{i=1}^{\infty} \frac{e^{\sigma G_i} \mathbf{X}_i}{R_i^b} \quad (8)$$

where

$$\mathbf{X}_i = k\alpha_i e^{j\phi_i} \left[\frac{D_i}{T} a_i e^{j\theta_i} + \left(1 - \frac{D_i}{T} \right) a'_i e^{j\theta'_i} \right], \quad (9)$$

and the distribution of \mathbf{W} is given by¹¹

$$\mathbf{W} \sim \mathcal{N}_c(0, N_0). \quad (10)$$

Since different interferers i transmit asynchronously and independently, the r.v.'s $\{\mathbf{X}_i\}_{i=1}^{\infty}$ are i.i.d.

The distribution of the aggregate interference \mathbf{Y} plays an important role in the performance evaluation of the probe link. In what follows, we characterize such distribution in two important scenarios: the \mathcal{P} -conditioned and unconditional cases.

B. \mathcal{P} -conditioned Interference Distribution

To derive the \mathcal{P} -conditioned distribution of the aggregate interference \mathbf{Y} in (8)-(9), we start with the results given in [27]. This work shows that in the case of Rayleigh fading, an expression of the form of (9) can be well approximated by a

¹⁰Boldface letters are used to denote complex quantities; for example, $\mathbf{Z} = Z_1 + jZ_2$.

¹¹We use $\mathcal{N}_c(0, \sigma^2)$ to denote a circularly symmetric (CS) complex Gaussian distribution, where the real and imaginary parts are i.i.d. $\mathcal{N}(0, \sigma^2/2)$.

circularly symmetric (CS) complex Gaussian r.v., such that

$$\mathbf{X}_i \sim \mathcal{N}_c(0, 2V_X), \quad V_X \triangleq \mathbb{V}\{X_{i,n}\}. \quad (11)$$

In [27], the validity of this approximation is justified both by analyzing the Kullback-Leibler divergence between the exact and approximated cases, as well as by comparing the corresponding error probabilities.¹² Then, conditioned on \mathcal{P} , the interference $\mathbf{Y} = \sum_{i=1}^{\infty} \frac{e^{\sigma G_i} \mathbf{X}_i}{R_i^b}$ becomes a sum of independent CS Gaussian r.v.'s and is therefore a CS Gaussian r.v. given by¹³

$$\mathbf{Y} \stackrel{\mathcal{P}}{\sim} \mathcal{N}_c(0, 2AV_X), \quad (12)$$

where A is defined as

$$A \triangleq \sum_{i=1}^{\infty} \frac{e^{2\sigma G_i}}{R_i^{2b}}. \quad (13)$$

Furthermore, V_X can be expressed after some algebra as

$$V_X = \frac{E}{3} + \frac{k^2}{6} \mathbb{E}\{a_i a'_i \cos(\theta_i - \theta'_i)\}, \quad i \geq 1, \quad (14)$$

where $E \triangleq k^2 \mathbb{E}\{a_i^2\}$ is the average symbol energy of each interfering node, measured 1 m away from the interferer [29].¹⁴ Because the r.v.'s $\{\mathbf{X}_i\}_{i=1}^{\infty}$ are i.i.d., V_X does not depend on i and is only a function of the interferers' signal constellation. For the case of equiprobable symbols and a constellation that is symmetric with respect to the origin of the IQ-plane¹⁵ (e.g., M -PSK and M -QAM), the second right-hand term in (14) vanishes and $V_X = E/3$.

Lastly, note that since A in (13) depends on the interferer positions \mathcal{P} (i.e., $\{R_i\}_{i=1}^{\infty}$ and $\{G_i\}_{i=1}^{\infty}$), it can be seen as a r.v. whose value is different for each realization of \mathcal{P} . Furthermore, Appendix B shows that r.v. A has a *skewed stable distribution* [30] given by¹⁶

$$A \sim \mathcal{S} \left(\alpha_A = \frac{1}{b}, \beta_A = 1, \gamma_A = \pi \lambda \mathcal{C}_{1/b}^{-1} e^{2\sigma^2/b^2} \right) \quad (15)$$

for $b > 1$, where \mathcal{C}_x is defined as

$$\mathcal{C}_x \triangleq \begin{cases} \frac{1-x}{\Gamma(2-x) \cos(\pi x/2)}, & x \neq 1, \\ \frac{2}{\pi}, & x = 1, \end{cases} \quad (16)$$

¹²We can obtain (11) following an alternative approach: if we consider that the interfering nodes are coded and operating close to capacity, then the signal transmitted by each interferer is Gaussian, such that $\mathbf{X}_i \sim \mathcal{N}_c(0, 2V_X)$ [28].

¹³We use $X \stackrel{\mathcal{Y}}{\sim}$ to denote the distribution of r.v. X conditional on \mathcal{Y} .

¹⁴Unless otherwise stated, we will simply refer to E as the "average symbol energy" of the interferers.

¹⁵A constellation is said to be *symmetric with respect to the origin* if for every constellation point $(x, y) \in \mathbb{R}^2$, the point $(-x, -y)$ also belongs to the constellation.

¹⁶We use $\mathcal{S}(\alpha, \beta, \gamma)$ to denote a real stable distribution with characteristic exponent $\alpha \in (0, 2]$, skewness $\beta \in [-1, 1]$, and dispersion $\gamma \in [0, \infty)$. The corresponding characteristic function is

$$\phi(w) = \begin{cases} \exp[-\gamma|w|^\alpha (1 - j\beta \operatorname{sign}(w) \tan \frac{\pi\alpha}{2})], & \alpha \neq 1, \\ \exp[-\gamma|w| (1 + j\frac{2}{\pi} \beta \operatorname{sign}(w) \ln |w|)], & \alpha = 1. \end{cases}$$

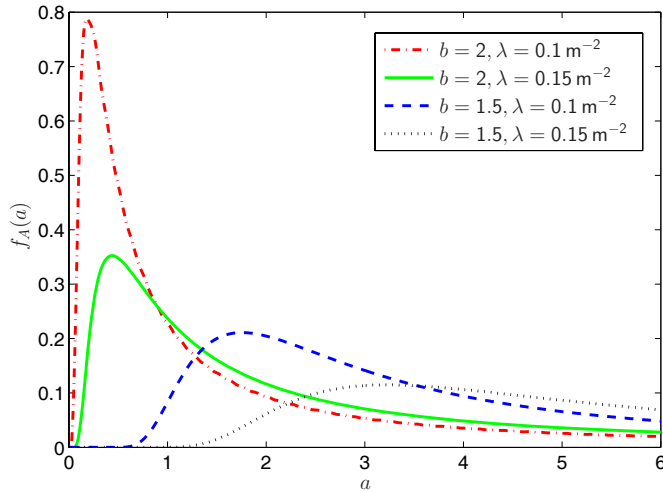


Fig. 3. P.d.f. of A for different amplitude loss exponents b and interferer densities λ ($\sigma_{\text{dB}} = 10$). Stable laws are a direct generalization of Gaussian distributions, and include other densities with heavier (algebraic) tails.

with $\Gamma(x) = \int_0^\infty t^{x-1} e^{-t} dt$ denoting the gamma function. This distribution is plotted in Fig. 3 for different b and λ .

C. Unconditional Interference Distribution

To derive the unconditional distribution¹⁷ of the aggregate interference \mathbf{Y} in (8)-(9), we can show that a sum of the form in (8) belongs to the class of *symmetric stable distributions* [30]. This is because $\{R_i\}_{i=1}^\infty$ are defined with respect to a spatial Poisson process, and $\{\mathbf{X}_i\}_{i=1}^\infty$ are i.i.d. and have a CS distribution. Specifically, Appendix C shows that \mathbf{Y} has a CS complex stable distribution given by¹⁸

$$\mathbf{Y} \sim \mathcal{S}_c \left(\alpha_{\mathbf{Y}} = \frac{2}{b}, \beta_{\mathbf{Y}} = 0, \right. \\ \left. \gamma_{\mathbf{Y}} = \pi \lambda \mathcal{C}_{2/b}^{-1} e^{2\sigma^2/b^2} \mathbb{E}\{|X_{i,n}|^{2/b}\} \right) \quad (17)$$

for $b > 1$. Using (5)-(6), we can further express $\mathbb{E}\{|X_{i,n}|^{2/b}\}$ in (17) as

$$\mathbb{E}\{|X_{i,n}|^{2/b}\} = k^{2/b} \mathbb{E}\{|\alpha_i|^{2/b}\} \\ \times \mathbb{E} \left\{ \underbrace{\left| a_i \frac{D_i}{T} \cos(\theta_i + \phi_i) + a'_i \left(1 - \frac{D_i}{T} \right) \cos(\theta'_i + \phi_i) \right|^{2/b}}_{\triangleq \chi(b)} \right\}. \quad (18)$$

For the particular case of Rayleigh fading, eq. (18) reduces to $\mathbb{E}\{|X_{i,n}|^{2/b}\} = k^{2/b} \Gamma(1 + \frac{1}{b}) \cdot \chi(b)$, where we have used the moment relation for the Rayleigh r.v. α_i [31]. Since different interferers i transmit asynchronously and independently, the parameter $\chi(b)$ does not depend on i and is only a function of the amplitude loss exponent b and the interferers' signal constellation. Table I provides some numerical values for $\mathbb{E}\{|X_{i,n}|^{2/b}\}$.

¹⁷Unconditional in the sense of being averaged over the positions \mathcal{P} .

¹⁸We use $\mathcal{S}_c(\alpha, \beta = 0, \gamma)$ to denote a CS complex stable distribution with characteristic exponent α and dispersion γ , and whose characteristic function is $\phi(\mathbf{w}) = \exp(-\gamma|\mathbf{w}|^\alpha)$. Furthermore, the corresponding real and imaginary components are both $\mathcal{S}(\alpha, \beta = 0, \gamma)$.

TABLE I
 $\mathbb{E}\{|X_{i,n}|^{2/b}\}$ FOR VARIOUS AMPLITUDE LOSS EXPONENTS b AND MODULATIONS, ASSUMING RAYLEIGH FADING. NOTE THAT FOR M -PSK MODULATIONS, THIS QUANTITY IS PROPORTIONAL TO $E^{1/b}$, WHERE E IS THE AVERAGE SYMBOL ENERGY OF THE INTERFERERS.

b	$\frac{\mathbb{E}\{ X_{i,n} ^{2/b}\}}{E^{1/b}}$	
	BPSK	QPSK
1.5	0.374	0.385
2	0.423	0.441
3	0.509	0.531
4	0.576	0.599

D. Discussion

The results of this section have to be interpreted carefully, because of the different types of conditioning involved. In the unconditional case, we let \mathcal{P} be random, i.e., we let $\{R_i\}_{i=1}^\infty$ be the random outcomes of an underlying spatial Poisson process, and $\{G_i\}_{i=1}^\infty$ be the random shadowing affecting each interferer. Then, the unconditional interference \mathbf{Y} is *exactly* stable-distributed and given by (17). We note that (17) and (18) hold for a broad class of fading distributions, in addition to Rayleigh fading. In the \mathcal{P} -conditioned case, the positions of the interferers are fixed. Then, A in (13) is also a fixed number, and the interference \mathbf{Y} is *approximately* CS Gaussian with total variance $2AV_X$, as given in (12).

IV. ERROR PROBABILITY

In the previous section, we determined the statistical distribution of the aggregate interference at the output of a conventional linear receiver. We now use such result to directly characterize of the error probability of the probe link, when subject to network interference and thermal noise, in both cases of slow and fast-varying \mathcal{P} .

A. Slow-varying Interferer Positions \mathcal{P}

In the quasi-static scenario of slow-varying \mathcal{P} , it is insightful to analyze the error probability conditioned on a given realization \mathcal{P} of the distances $\{R_i\}_{i=1}^\infty$ and shadowing $\{G_i\}_{i=1}^\infty$ associated with the interferers, as well as on the shadowing G_0 of the probe transmitter. We denote this conditional symbol error probability by $P_e(G_0, \mathcal{P})$.¹⁹

To derive the conditional error probability, we employ the results of Section III-B for the \mathcal{P} -conditioned distribution of the aggregate interference \mathbf{Y} . Specifically, using (10) and (12), the received signal \mathbf{Z} in (7) can be rewritten as

$$\mathbf{Z} = \frac{k\alpha_0 e^{\sigma G_0}}{r_0^b} a_0 e^{j\theta_0} + \widetilde{\mathbf{W}}, \quad (19)$$

where

$$\widetilde{\mathbf{W}} = \mathbf{Y} + \mathbf{W} \stackrel{\mathcal{P}}{\sim} \mathcal{N}_c(0, 2AV_X + N_0). \quad (20)$$

Our framework has thus reduced the analysis to a Gaussian scenario, where the combined noise $\widetilde{\mathbf{W}}$ is Gaussian when conditioned on the location of the interferers.

¹⁹The notation $P_e(X, Y)$ is used as a shorthand for $\mathbb{P}\{\text{error}|X, Y\}$.

The corresponding error probability $P_e(G_0, \mathcal{P})$ can be found by taking the well-known error probability expressions for coherent detection of linear modulations in the presence of AWGN and fast fading [32]–[35], but using $2AV_X + N_0$ instead of N_0 for the total noise variance. Note that this substitution is valid for any linear modulation, allowing the traditional results to be extended to include the effect of network interference. For the particular case where the probe transmitter employs an arbitrary signal constellation in the IQ-plane and the fading is Rayleigh-distributed, the conditional symbol error probability $P_e(G_0, \mathcal{P})$ is given by

$$P_e(G_0, \mathcal{P}) = \sum_{k=1}^M p_k \sum_{l \in \mathcal{B}_k} \frac{1}{2\pi} \int_0^{\phi_{k,l}} \left(1 + \frac{w_{k,l}}{4 \sin^2(\theta + \psi_{k,l})} \eta_A \right)^{-1} d\theta, \quad (21)$$

where

$$\eta_A = \frac{e^{2\sigma G_0} E_0}{r_0^{2b} (2AV_X + N_0)} \quad (22)$$

is the received signal-to-interference-plus-noise ratio (SINR), averaged over the fast fading; M is the constellation size; $\{p_k\}_{k=1}^M$ are the symbol probabilities; \mathcal{B}_k , $\phi_{k,l}$, $w_{k,l}$, and $\psi_{k,l}$ are the parameters that describe the geometry of the constellation (see Fig. 4); $E_0 \triangleq k^2 \mathbb{E}\{a_0^2\}$ is the average symbol energy of the probe transmitter, measured 1 m away from the transmitter; A is defined in (13) and distributed according to (15); and V_X is given in (14). When the probe transmitter employs M -PSK and M -QAM modulations with equiprobable symbols, eq. (21) is equivalent to²⁰

$$P_e^{\text{MPSK}}(G_0, \mathcal{P}) = \mathcal{I}_A \left(\frac{M-1}{M} \pi, \sin^2 \left(\frac{\pi}{M} \right) \right) \quad (23)$$

and

$$P_e^{\text{MQAM}}(G_0, \mathcal{P}) = 4 \left(1 - \frac{1}{\sqrt{M}} \right) \cdot \mathcal{I}_A \left(\frac{\pi}{2}, \frac{3}{2(M-1)} \right) - 4 \left(1 - \frac{1}{\sqrt{M}} \right)^2 \cdot \mathcal{I}_A \left(\frac{\pi}{4}, \frac{3}{2(M-1)} \right), \quad (24)$$

respectively, where the integral $\mathcal{I}_A(x, g)$ is given by

$$\mathcal{I}_A(x, g) = \frac{1}{\pi} \int_0^x \left(1 + \frac{g}{\sin^2 \theta} \eta_A \right)^{-1} d\theta. \quad (25)$$

In the general expression given in (21) and (22), the network interference is accounted for by the term $2AV_X$, where A depends on the spatial distribution of the interferers and propagation characteristics of the medium, while V_X depends on the interferer transmission characteristics. Since $2AV_X$ simply adds to N_0 , we conclude that the effect of the interference on the error probability is simply an increase in the noise level, a fact which is intuitively satisfying. Furthermore, note that the modulation of the interfering nodes only affects the term V_X , while the (possibly different) modulation of the probe transmitter affects the *type* of error probability expression, leading to forms such as (23) or (24).

²⁰In this paper, we implicitly assume that M -QAM employs a square signal constellation with $M = 2^n$ points (n even).

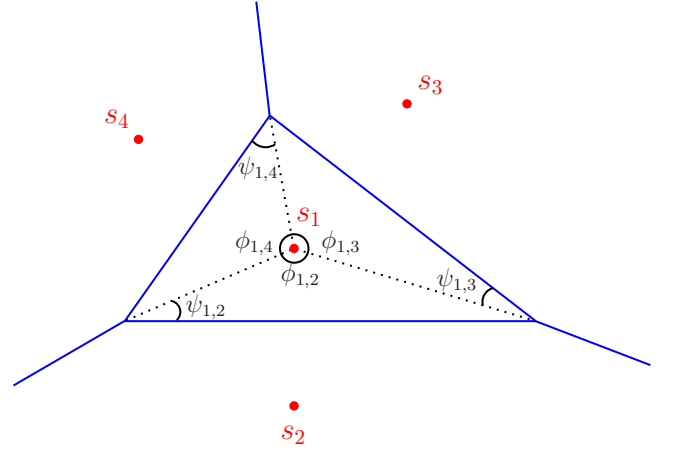


Fig. 4. Typical decision region associated with symbol s_1 . In general, for a constellation with signal points $s_k = |s_k|e^{j\xi_k}$ and $\zeta_k = \frac{|s_k|^2}{\mathbb{E}\{|s_k|^2\}}$, $k = 1 \dots M$, four parameters are required to compute the error probability: $\phi_{k,l}$ and $\psi_{k,l}$ are the angles that describe the decision region corresponding to s_k (as depicted); \mathcal{B}_k is the set consisting of the indexes for the signal points that share a decision boundary with s_k (in the example, $\mathcal{B}_1 = \{2, 3, 4\}$); and $w_{k,l} = \zeta_k + \zeta_l - 2\sqrt{\zeta_k \zeta_l} \cos(\xi_k - \xi_l)$.

In our quasi-static model, the conditional error probability in (21) is seen to be a function of the slow-varying user positions and shadowing (i.e., G_0 and \mathcal{P}). Since these quantities are random, the error probability itself is a r.v. Then, with some probability, G_0 and \mathcal{P} are such that the error probability of the probe link is above some target p^* . In this case, the system is said to be *in outage*, and the error outage probability is

$$P_{\text{out}}^e = \mathbb{P}_{G_0, \mathcal{P}} \{ P_e(G_0, \mathcal{P}) > p^* \}, \quad (26)$$

In the case of slow-varying user positions, the error outage probability is a more meaningful metric than the error probability averaged over \mathcal{P} .

B. Fast-varying Interferer Positions \mathcal{P}

In the dynamic scenario of fast-varying \mathcal{P} , it is insightful to average the error probability over all possible realizations of interferer positions \mathcal{P} . We denote this average symbol error probability by $P_e(G_0)$. Note that we choose not to average out the shadowing G_0 affecting the probe transmitter, since we consider the probe transmitter node to be immobile at a deterministic distance r_0 from the probe receiver, and thus G_0 is slow-varying.

To derive the average error probability, we use the decomposition property of stable r.v.'s [30], which allows \mathbf{Y} in (17) to be decomposed as

$$\mathbf{Y} = \sqrt{B} \mathbf{G}, \quad (27)$$

where B and \mathbf{G} are independent r.v.'s, and

$$B \sim \mathcal{S} \left(\alpha_B = \frac{1}{b}, \beta_B = 1, \gamma_B = \cos \frac{\pi}{2b} \right) \quad (28)$$

$$\mathbf{G} \sim \mathcal{N}_c(0, 2V_G), \quad V_G = 2e^{2\sigma^2/b} \left(\pi \lambda \mathcal{C}_{2/b}^{-1} \mathbb{E}\{|X_{i,n}|^{2/b}\} \right)^b, \quad (29)$$

with $\mathbb{E}\{|X_{i,n}|^{2/b}\}$ given in (18). Conditioning on the r.v. B , we then use (10) and (27) to rewrite the aggregate received

signal \mathbf{Z} in (7) as

$$\mathbf{Z} = \frac{k\alpha_0 e^{\sigma G_0}}{r_0^b} a_0 e^{j\theta_0} + \widetilde{\mathbf{W}},$$

where

$$\widetilde{\mathbf{W}} = \sqrt{B}\mathbf{G} + \mathbf{W} \stackrel{|B|}{\sim} \mathcal{N}_c(0, 2BV_G + N_0). \quad (30)$$

Again, our framework has reduced the analysis to a Gaussian scenario, where the combined noise $\widetilde{\mathbf{W}}$ is a Gaussian r.v. Note that this result was derived without resorting to any approximations – in particular, the Gaussian approximation of (11) was not needed here. We merely used the decomposition property of symmetric stable r.v.'s.

The corresponding error probability $P_e(G_0)$ can be found by taking the error expressions for coherent detection in the presence of AWGN and fast fading, then using $2BV_G + N_0$ instead of N_0 for the total noise variance, and lastly (unlike in Section IV-A) averaging over the r.v. B . For the particular case where the probe transmitter employs an arbitrary signal constellation in the IQ-plane and the fading is Rayleigh-distributed, the average symbol error probability $P_e(G_0)$ is given by

$$P_e(G_0) = \sum_{k=1}^M p_k \sum_{l \in \mathcal{B}_k} \frac{1}{2\pi} \times \int_0^{\phi_{k,l}} \mathbb{E}_B \left\{ \left(1 + \frac{w_{k,l}}{4 \sin^2(\theta + \psi_{k,l})} \eta_B \right)^{-1} \right\} d\theta, \quad (31)$$

where

$$\eta_B = \frac{e^{2\sigma G_0} E_0}{r_0^{2b} (2BV_G + N_0)}; \quad (32)$$

B is distributed according to (28); V_G is given in (29); and the other parameters have the same meaning as in Section IV-A. When the probe transmitter employs M -PSK and M -QAM modulations with equiprobable symbols, eq. (31) is equivalent to

$$P_e^{\text{MPSK}}(G_0) = \mathcal{I}_B\left(\frac{M-1}{M}\pi, \sin^2\left(\frac{\pi}{M}\right)\right) \quad (33)$$

and

$$P_e^{\text{MQAM}}(G_0) = 4 \left(1 - \frac{1}{\sqrt{M}}\right) \cdot \mathcal{I}_B\left(\frac{\pi}{2}, \frac{3}{2(M-1)}\right) - 4 \left(1 - \frac{1}{\sqrt{M}}\right)^2 \cdot \mathcal{I}_B\left(\frac{\pi}{4}, \frac{3}{2(M-1)}\right), \quad (34)$$

respectively, where the integral $\mathcal{I}_B(x, g)$ is given by

$$\mathcal{I}_B(x, g) = \frac{1}{\pi} \int_0^x \mathbb{E}_B \left\{ \left(1 + \frac{g}{\sin^2 \theta} \eta_B \right)^{-1} \right\} d\theta. \quad (35)$$

C. Discussion

The results derived in Sections IV-A and IV-B provide insights into the dependence of the error performance on the density λ and the average symbol energy E of the interfering nodes. Recall from (21) that the error probability $P_e(G_0, \mathcal{P})$ implicitly depends on parameters λ and E through the product AV_X in the denominator of η_A in (22). This is because the dispersion parameter γ_A of the stable r.v. A depends on λ according to (15), and V_X is proportional to E as in (14). The

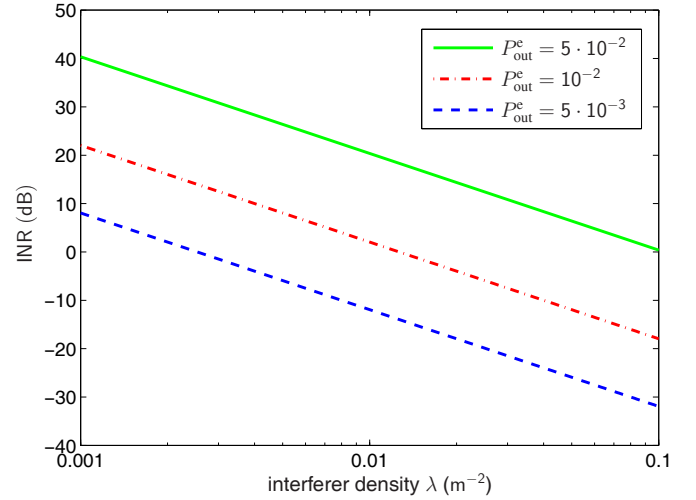


Fig. 5. INR – λ curves of constant P_{out}^e , for the case of slow-varying interferer positions \mathcal{P} (BPSK, SNR = 40 dB, $b = 2$, $r_0 = 1$ m, $\sigma_{\text{dB}} = 10$, $p^* = 10^{-2}$). The INR is defined as $\text{INR} = E/N_0$. Clearly, for a fixed error performance, there is a tradeoff between the density and energy of the interferers: if the INR (or, equivalently, E) increases, λ must decrease, and vice-versa, to maintain the same outage probability.

dependence on λ can be made evident by using the scaling property of stable r.v.'s [30] to write $AV_X = \lambda^b \tilde{A}V_X$, where \tilde{A} is a normalized version of A , independent of λ . We thus conclude that the interference term AV_X is proportional to $\lambda^b E$, where $b > 1$. Clearly, the error performance degrades faster with an increase in the *density* of interferers than with an increase in their *transmitted power*. The tradeoff between E and λ for a fixed error performance is illustrated in Fig. 5.

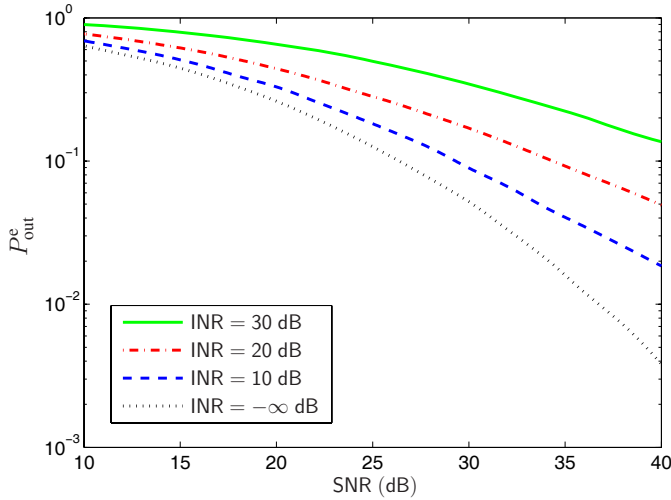
D. Numerical Results

Figs. 6 and 7 quantify the average and outage probabilities for several scenarios, showing their dependence on various parameters involved, such as the signal-to-noise ratio $\text{SNR} = E_0/N_0$, interference-to-noise ratio $\text{INR} = E/N_0$, amplitude loss exponent b , interferer spatial density λ , and link length r_0 .

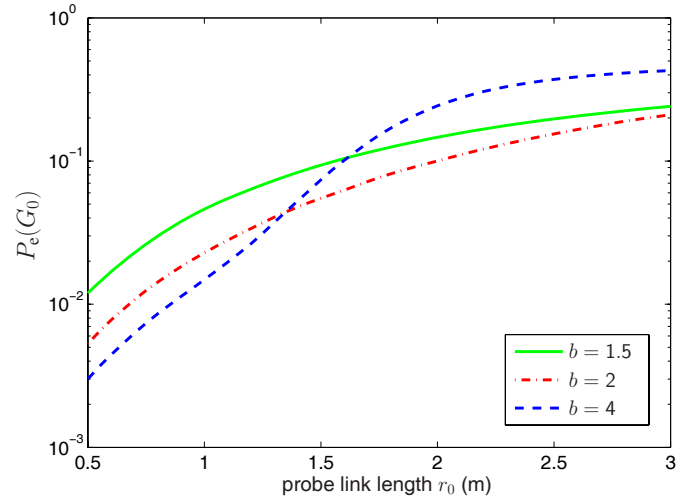
The plots of P_{out}^e and $P_e(G_0)$ presented here are of semi-analytical nature. Specifically, we resort to a hybrid method where we employ the analytical results given in (21)-(26) and (31)-(35), and perform a Monte Carlo simulation with respect to the stable r.v.'s (i.e., A and B), according to [36]. We emphasize that the expressions derived in this paper completely eliminate the need for simulation of the interferers' position and waveforms in the network, in order to obtain the error performance.

For illustration purposes, we consider that all terminals (i.e., the probe transmitter and interfering nodes) use BPSK modulation. We analyze both cases of slow and fast-varying interferer positions \mathcal{P} , concurrently with the following two different scenarios:

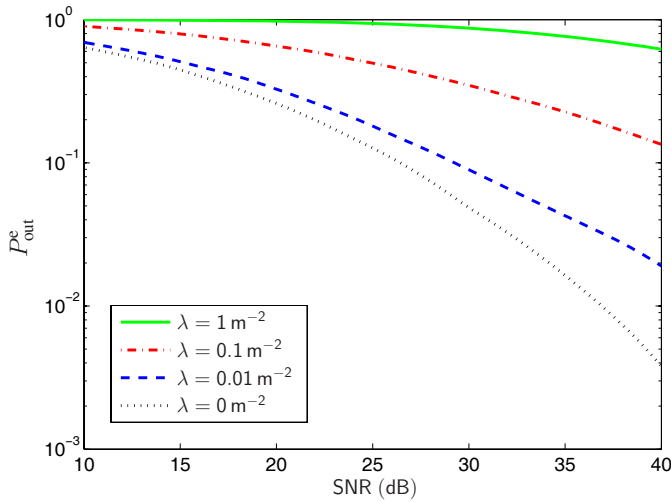
- 1) *Heterogeneous network*: The probe transmitter is allowed to use an arbitrary power $P_0 = E_0/T$, not necessarily equal to the common power of the interfering nodes $P = E/T$, and hence $\text{SNR} \neq \text{INR}$ in general. This scenario is useful when the goal is to evaluate the impact of aggregate interference from a large number



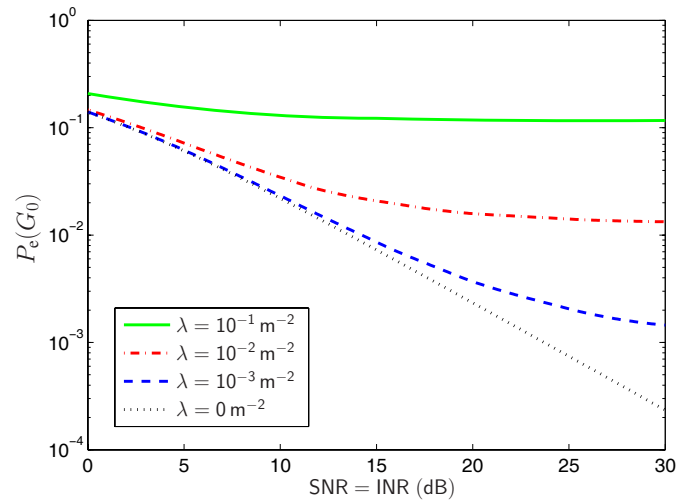
(a) P_{out}^e versus the SNR of the probe link, for various interference-to-noise ratios INR (BPSK, $b = 2$, $\lambda = 0.01 \text{ m}^{-2}$, $r_0 = 1 \text{ m}$, $\sigma_{\text{dB}} = 10$, $p^* = 10^{-2}$).



(a) $P_e(G_0)$ versus the length r_0 of the probe link, for various signal loss exponents b (BPSK, $G_0 = 0$, SNR = INR = 20 dB, $\lambda = 0.01 \text{ m}^{-2}$, $\sigma_{\text{dB}} = 10$).



(b) P_{out}^e versus the SNR of the probe link, for various interferer spatial densities λ (BPSK, INR = 10 dB, $b = 2$, $r_0 = 1 \text{ m}$, $\sigma_{\text{dB}} = 10$, $p^* = 10^{-2}$).



(b) $P_e(G_0)$ versus the SNR, for various interferer densities λ (BPSK, $G_0 = 0$, $b = 3$, $r_0 = 1 \text{ m}$, $\sigma_{\text{dB}} = 10$).

Fig. 6. Error outage probability plots for a heterogeneous network (where $\text{SNR} \neq \text{INR}$ in general) and slow-varying interferer positions \mathcal{P} . Since \mathcal{P} is slow-varying, the meaningful performance metric is the outage probability P_{out}^e given in (26).

Fig. 7. Average error probability plots for a homogeneous network (where $\text{SNR} = \text{INR}$) and fast-varying interferer positions \mathcal{P} . Since \mathcal{P} is fast-varying, the meaningful performance metric is the average error probability $P_e(G_0)$ given in (31). For simplicity, we use $G_0 = 0$ in these plots (no shadowing on the probe link).

of identical secondary users (e.g., cognitive-radio terminals) on the performance of a primary link.

- 2) *Homogeneous network*: The probe transmitter and interfering nodes all use the same power, and thus $\text{SNR} = \text{INR}$. This may correspond to a sensor network scenario, where there is a large number of indistinguishable, spatially scattered nodes with similar transmission characteristics. In such a case, the goal is to evaluate the impact of the aggregate network self-interference on the performance of each sensor node.

For the heterogeneous case depicted in Fig. 6, we conclude that the error performance deteriorates as λ or INR increase, for a fixed SNR. This is expected because as the density or the transmitted energy of the interferers increase, the aggregate interference at the probe receiver becomes stronger. Note, however, that in the homogeneous case where $\text{SNR} = \text{INR}$, the error performance improves as we increase the common

transmitted power P of the nodes (or equivalently, the SNR), although the gains become marginally small as $P \rightarrow \infty$ (see Fig. 7(b)). This happens because in the interference-limited regime where $\text{SNR} = \text{INR} \gg 1$, the noise term N_0 in (22) or (32) becomes irrelevant, and so the SNR in the numerator cancels with the INR in the denominator, making the performance independent of the transmitted power P .

The effect of the amplitude loss exponent b on the error performance, on the other hand, is non-trivial. As illustrated in Fig. 7(a), an increase in b may degrade or improve the performance, depending on the distance r_0 and other parameters. This is because b simultaneously affects both the received signal of interest and the aggregate interference – the former, through the term $1/r_0^b$; and the latter, through α_A and γ_A in (15), or through α_B , γ_B , and V_G in (28) and (29).

$$\begin{aligned}
Y_1 &= \int_0^T Y(t)\psi_1(t)dt \\
&= \sum_{i=1}^{\infty} \int_0^T \frac{k\alpha_i e^{\sigma G_i}}{R_i^b} \left[\sqrt{\frac{2}{T}} a_i \cos(2\pi f_c t + \theta_i + \phi_i) u(D_i - t) \right. \\
&\quad \left. + \sqrt{\frac{2}{T}} a'_i \cos(2\pi f_c t + \theta'_i + \phi_i) u(t - D_i) \right] \times \sqrt{\frac{2}{T}} \cos(2\pi f_c t) dt \\
&= \sum_{i=1}^{\infty} \frac{2}{T} \frac{k\alpha_i e^{\sigma G_i}}{R_i^b} \left[\frac{a_i}{2} \int_0^{D_i} \cos(\theta_i + \phi_i) dt + \frac{a_i}{2} \underbrace{\int_0^{D_i} \cos(4\pi f_c t + \theta_i + \phi_i) dt}_{\approx 0 \text{ for } f_c T \gg 1} \right. \\
&\quad \left. + \frac{a'_i}{2} \int_{D_i}^T \cos(\theta'_i + \phi_i) dt + \frac{a'_i}{2} \underbrace{\int_{D_i}^T \cos(4\pi f_c t + \theta'_i + \phi_i) dt}_{\approx 0 \text{ for } f_c T \gg 1} \right] \\
&= \sum_{i=1}^{\infty} \frac{e^{\sigma G_i} X_{i,1}}{R_i^b}
\end{aligned} \tag{36}$$

V. CONCLUSIONS

This paper introduces a mathematical model for communication subject to network interference and noise. The interferers are scattered according to a spatial Poisson process, and are operating asynchronously in a wireless environment subject to path loss, shadowing, and multipath fading. We show that the aggregate network interference at the output of a linear receiver is related to a *skewed stable distribution* when conditioned on the positions of interferers, and to a *symmetric stable distribution* in the unconditional case. We characterize the error performance for the cases of slow and fast-varying interferers, in terms of outage and average error probabilities, respectively. These expressions are valid for any linear modulation scheme. We then quantify these metrics as a function of various important system parameters, such as the SNR, INR, path loss exponent, and spatial density of the interferers. In Part II of the paper [11], we characterize the capacity of the link when subject to both network interference and noise, and derive the spectrum of the aggregate interference at any location in the plane. Lastly, we put forth the concept of spectral outage probability, a new characterization of the aggregate interference generated by communicating nodes in a wireless network.

APPENDIX A

DERIVATION OF THE INTERFERENCE REPRESENTATION IN (4)-(6)

To obtain the desired representation, we project $Y(t)$ onto the basis function $\psi_1(t) = \sqrt{2/T} \cos(2\pi f_c t)$ as given in (36) at the top of this page, where

$$X_{i,1} = k\alpha_i \left[a_i \frac{D_i}{T} \cos(\theta_i + \phi_i) + a'_i \left(1 - \frac{D_i}{T} \right) \cos(\theta'_i + \phi_i) \right].$$

The signal $Y(t)$ can be projected onto the basis function $\psi_2(t) = -\sqrt{2/T} \sin(2\pi f_c t)$ in an entirely analogous

way, leading to

$$Y_2 = \sum_{i=1}^{\infty} \frac{e^{\sigma G_i} X_{i,2}}{R_i^b},$$

where

$$X_{i,2} = k\alpha_i \left[a_i \frac{D_i}{T} \sin(\theta_i + \phi_i) + a'_i \left(1 - \frac{D_i}{T} \right) \sin(\theta'_i + \phi_i) \right].$$

This completes the derivation.

APPENDIX B

DERIVATION OF THE DISTRIBUTION OF A

To derive the distribution of A given in (15), we start with the following theorem.

Theorem B.1: Let $\{\tau_i\}_{i=1}^{\infty}$ denote the arrivals of a one-dimensional Poisson process with rate λ ; let $\{W_i\}_{i=1}^{\infty}$ be a sequence of nonnegative i.i.d. r.v.'s, independent of the sequence $\{\tau_i\}$ and satisfying $\mathbb{E}\{|W_i|^\alpha\} < \infty$. If $0 < \alpha < 1$, then

$$\sum_{i=1}^{\infty} \frac{W_i}{\tau_i^{1/\alpha}} \stackrel{\text{a.s.}}{\sim} \mathcal{S}(\alpha, \beta = 1, \gamma = \lambda \mathcal{C}_\alpha^{-1} \mathbb{E}\{|W_i|^\alpha\}),$$

where \mathcal{C}_α is defined in (16).

Proof: See [30]. \square

If an homogeneous Poisson point process *in the plane* has spatial density λ , and R_i denotes the distance of node i to the origin, then the sequence $\{R_i^2\}_{i=1}^{\infty}$ represents Poisson arrivals *on the line* with the constant arrival rate $\pi\lambda$. This can be easily shown by mapping the spatial Poisson process from Cartesian into polar coordinates, and then applying the mapping theorem [12]. Using this fact, we can then apply the

above theorem to (13) and write

$$A = \sum_{i=1}^{\infty} \frac{e^{2\sigma G_i}}{R_i^{2b}} = \sum_{i=1}^{\infty} \frac{\overbrace{e^{2\sigma G_i}}^{W_i}}{\underbrace{(R_i^2)^b}_{\tau_i}} \stackrel{\text{a.s.}}{\sim} \mathcal{S} \left(\alpha = \frac{1}{b}, \beta = 1, \gamma = \pi \lambda \mathcal{C}_{1/b}^{-1} \mathbb{E}\{|e^{2\sigma G_i}|^{1/b}\} \right) \quad (37)$$

for $b > 1$. Using the moment property of log-normal r.v.'s, i.e., $\mathbb{E}\{e^{kG}\} = e^{k^2/2}$ for $G \sim \mathcal{N}(0, 1)$, eq. (37) simplifies to

$$A \stackrel{\text{a.s.}}{\sim} \mathcal{S} \left(\alpha = \frac{1}{b}, \beta = 1, \gamma = \pi \lambda \mathcal{C}_{1/b}^{-1} e^{2\sigma^2/b^2} \right)$$

for $b > 1$. This is the result in (15) and the derivation is complete.

APPENDIX C

DERIVATION OF THE DISTRIBUTION OF \mathbf{Y}

To derive the distribution of \mathbf{Y} given in (17), we start with the following theorem.

Theorem C.1: Let $\{\tau_i\}_{i=1}^{\infty}$ denote the arrivals of a one-dimensional Poisson process with rate λ ; let $\{\mathbf{Z}_i\}_{i=1}^{\infty}$ be a sequence of CS i.i.d. complex r.v.'s $\mathbf{Z}_i = Z_{i,1} + jZ_{i,2}$, independent of the sequence $\{\tau_i\}$ and satisfying $\mathbb{E}\{|\mathbf{Z}_i|^\alpha\} < \infty$. If $0 < \alpha < 2$, then

$$\sum_{i=1}^{\infty} \frac{\mathbf{Z}_i}{\tau_i^{1/\alpha}} \stackrel{\text{a.s.}}{\sim} \mathcal{S}_c \left(\alpha, \beta = 0, \gamma = \lambda \mathcal{C}_\alpha^{-1} \mathbb{E}\{|\mathbf{Z}_i|^\alpha\} \right),$$

where \mathcal{C}_α is defined in (16).

Proof: See [30]. For an alternative proof based on the influence function method, see [37]. \square

Using the Poisson mapping theorem as in Appendix B, we can apply the above theorem to (8) and write

$$\mathbf{Y} = \sum_{i=1}^{\infty} \frac{e^{\sigma G_i} \mathbf{X}_i}{R_i^b} = \sum_{i=1}^{\infty} \frac{\overbrace{e^{\sigma G_i} \mathbf{X}_i}^{\text{CS i.i.d.}}}{\underbrace{(R_i^2)^{b/2}}_{\tau_i}} \stackrel{\text{a.s.}}{\sim} \mathcal{S}_c \left(\alpha = \frac{2}{b}, \beta = 0, \gamma = \pi \lambda \mathcal{C}_{2/b}^{-1} \mathbb{E}\{|e^{\sigma G_i} X_{i,n}|^{2/b}\} \right) \quad (38)$$

for $b > 1$. Note that \mathbf{X}_i , whose expression is given in (9), is CS due to the uniform phase ϕ_i . As a result, $e^{\sigma G_i} \mathbf{X}_i$ is also CS. Using the moment property of log-normal r.v.'s, i.e., $\mathbb{E}\{e^{kG}\} = e^{k^2/2}$ with $G \sim \mathcal{N}(0, 1)$, eq. (38) simplifies to

$$\mathbf{Y} \stackrel{\text{a.s.}}{\sim} \mathcal{S}_c \left(\alpha = \frac{2}{b}, \beta = 0, \gamma = \pi \lambda \mathcal{C}_{2/b}^{-1} e^{2\sigma^2/b^2} \mathbb{E}\{|X_{i,n}|^{2/b}\} \right)$$

for $b > 1$. This is the result in (17) and the derivation is complete.

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