MIMO Systems with Antenna Selection

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Multiple-input–multiple-output (MIMO) wireless systems are those that have multiple antenna elements at both the transmitter and receiver [1]. They were first investigated by computer simulations in the 1980s [2], and later papers explored them analytically [3], [4]. Since that time, interest in MIMO systems has exploded. They are now being used for third-generation cellular systems (W-CDMA) and are discussed for future high-performance modes of the highly successful IEEE 802.11 standard for wireless local area networks. MIMO-related topics also occupy a considerable part of today's academic communications research.

The multiple antennas in MIMO systems can be exploited in two different ways. One is the creation of a highly effective antenna diversity system; the other is the use of the multiple antennas for the transmission of several parallel data streams to increase the capacity of the system.

Antenna diversity is used in wireless systems to combat the effects of fading. If multiple, independent copies of the same signal are available, we can combine them into a *total signal* with high quality—even if *some* of the copies exhibit low quality. Antenna diversity at the receiver is well known and has been studied for more than 50 years. The different signal copies are linearly combined, i.e., weighted and added. The resulting signal at the combiner output can then be demodulated and decoded in the usual way. The optimum weights for this combining are matched to the wireless channel [maximum ratio combining (MRC)]. If we have N receive antenna elements, the diversity order, which describes the effectiveness of diversity in avoiding

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deep fades, is N; in other words, the diversity order is related to the slope of the signal-to-noise ratio (SNR) distribution at the combiner output. The multiple antennas also increase the *average* SNR seen at the combiner output. The study of transmit diversity is much more recent, starting in the 1990s. When the channel is known to the transmitter, we can again "match" the multiple transmitted signal copies to the channel, resulting in the same gains as for receiver diversity. If the channel is unknown at the transmitter, other strategies, like delay diversity or space-time-coding, have to be used. In that case, we can gain high diversity order, but not improvement of average SNR. The logical next step is the combination of transmit and receive diversity. It has been demonstrated that with N_t transmit and N_r receive antennas, a diversity order of $N_t N_r$ can be achieved [5]. A MIMO system can thus be used for a high-quality transmission of a single data stream even in challenging environments.

An alternative way of exploiting the multiple antenna elements is the so-called "spatial multiplexing" [6] or "BLAST" [7] approach. The principle of this approach is sketched in Figure 1. Different data streams are transmitted (in parallel) from the different transmit antennas. The multiple receive antenna elements are used for separating the different data streams at the receiver. We have N_r combinations of the N_t transmit signals. If the channel is well-behaved, so that the N_r received signals represent linearly independent combinations, we can recover the transmit signals as long as $N_t \leq N_r$. The advantage of this method is that the data rate can be increased by a factor N_t without requiring more spectrum! In this article, we will mostly discuss the information-theoretic capacity, i.e., the data rate that can be transmitted over a channel without errors if ideal coding is used. Practical schemes, like layered space-time (ST) receiver structures [8]–[10] combined with space-time codes [11] allow us to approach these capacity limits.

Antenna Selection for MIMO

Regardless of the use as diversity or spatial multiplexing system, the main drawback of any MIMO system is the increased complexity, and, thus, cost. While additional antenna elements (patch or dipole antennas) are usually inexpensive, and the additional digital signal processing becomes ever cheaper, the RF elements are expensive and do not follow Moore's law. MIMO systems with N_t transmit and N_r receive antennas require N_t (N_r) complete RF chains at the transmitter, and the receiver, respectively, including low-noise amplifiers, downconverters, and analog-to-digital converters.

Optimum selection algorithms have a complexity $\binom{N}{L}$. However, fast selection algorithms do exist that have much lower complexity and perform almost as well as full-complexity systems.

Due to this reason, there is now great interest in so-called hybrid-selection schemes, where the "best" L out of N antenna signals are chosen (either at one or both link ends), downconverted, and processed. This reduces the number of required RF chains from N to L, and, thus, leads to significant savings. The savings come at the price of a (usually small) performance loss compared to the full-complexity system. In the case that the multiple antennas are used for diversity purposes, the approach is called "hybrid selection/maximum-ratio-combining" (H-S/MRC), or sometimes also "generalized selection combining" [12]–[14]; if they are used for spatial multiplexing, the scheme is called "hybrid selection/MIMO" (H-S/MIMO) [15]. In this article, we describe the performance that can be achieved with such a system, how the "best" antennas can be selected in an efficient manner, and how non-idealities affect the performance.

Notation

In this article, a vector is denoted by an arrow, $\vec{x}$, a matrix by underline $\underline{A}$. Superscript * denotes complex conjugation; superscript † denotes the Hermitian transpose.

System Model

Figure 2 shows the generic system that we are considering. A bit stream is sent through a vector encoder and

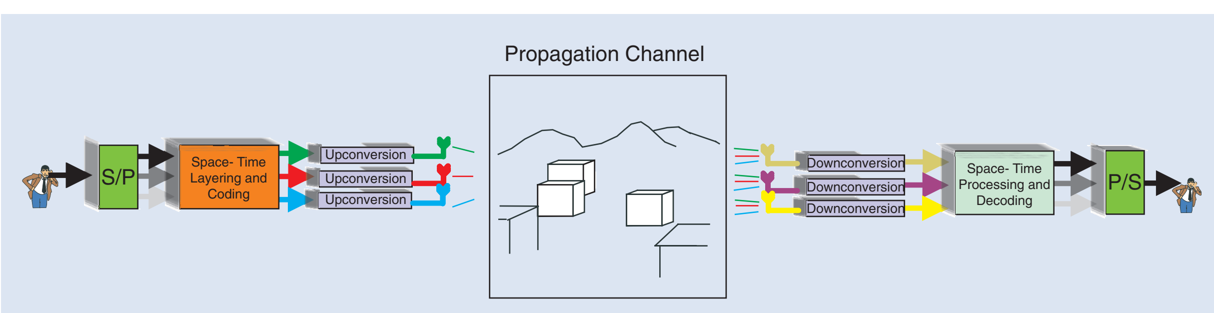

Figure 1. *Principle of spatial multiplexing.*

modulator. This encoder converts a single bit stream into L_t parallel streams of complex symbols. These streams can have all the same information (e.g., for a simple transmit diversity system with channel knowledge), can all have independent symbol streams (e.g., in V-BLAST spatial multiplexing), or have partially correlated data streams. Subsequently, a multiplexer switches the modulated signals to the best L_t out of N_t available antenna branches. For each selected branch, the signal is multiplied by a complex weight u whose actual value depends on the current channel realization. If the channel is unknown at the transmitter, all weights are set to unity.

Antenna selection retains the diversity degree, compared to the full-complexity system, for both linear diversity systems with complete channel knowledge and space-time coded systems.

In a realistic system, the signals are subsequently upconverted to passband, amplified by a power amplifier, and filtered. For our model, we omit these stages, as well as their corresponding stages at the receiver, and treat the whole problem in equivalent baseband. Note, however, that exactly these stages are the most expensive and make the use of antenna selection desirable.

Next, the signal is sent over a quasistatic flat-fading channel. We denote the $N_r \times N_t$ matrix of the channel as $\underline{H}$. The entry with $h_{k,m}$ denotes the (complex) attenuation between the mth transmit and the kth receive antenna. The output of the channel is polluted by additive white Gaussian noise, which is assumed to be independent at all receiving antenna elements. At the receiver, the best L_r of the available N_r antenna elements are selected, and downconverted for further processing (note that only L_r receiver chains are required). This further processing can consist of weighting with complex weights $\vec{w}^*$ and linear combining (if the transmitter uses simple transmit diversity), or space-time-processing and -decoding.

Unless otherwise stated, we assume in the following that

1) The fading at the different antenna elements is assumed to be independent identically distributed (i.i.d.) Rayleigh fading. This is fulfilled if the directions of the multipath components at the transmitter and receiver are approximately uniform, and/or the antenna elements are spaced far apart from each other [16].
2) The fading is assumed to be frequency flat. This is fulfilled if the coherence bandwidth of the channel is significantly larger than the transmission bandwidth.
3) We assume that the receiver has perfect knowledge of the channel. For the transmitter, we will analyze both cases where the transmitter has no channel knowledge, and where it has perfect channel knowledge.
4) When talking about capacity, we also assume that the channel is quasistatic. By quasistatic, we mean that the coherence time of the channel is so long that "almost infinitely" many bits can be transmitted within this time. Thus, each channel realization is associated with a (Shannon—AWGN) capacity value. The capacity thus becomes a random variable, described by its cumulative distribution function (cdf).

The input–output relationship can thus be written as

$$\vec{y} = \underline{H}\vec{s} + \vec{n} = \vec{x} + \vec{n} \qquad (1)$$

where $\vec{s}$ is the transmit signal vector, and $\vec{n}$ is the noise vector.

Performance of Single Input—Multiple Output (SIMO) Systems

In order to explain some of the principles, we first consider the case where there is only a single transmit antenna, and antenna selection is used at the receiver. In that case, the multiple antennas can be used only for H-S/MRC diversity (no parallel data streams are possible). It is optimum to select the L out of N antennas that provide the largest SNR at each instant. These antennas

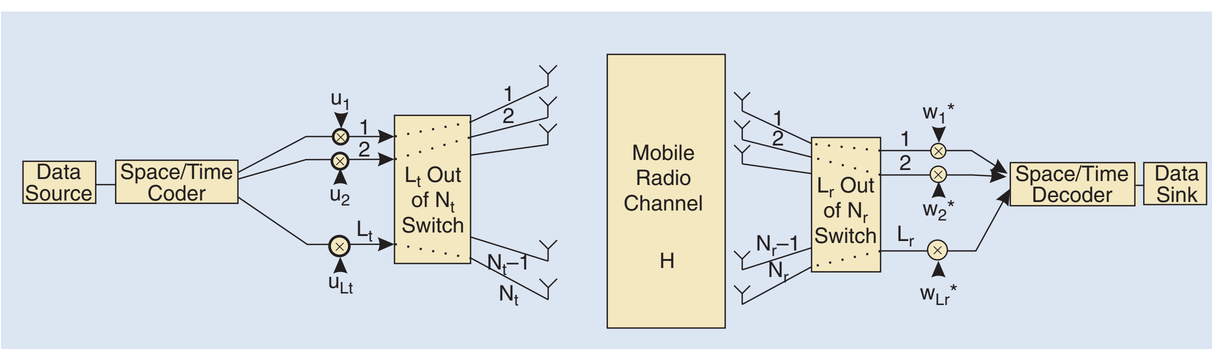

Figure 2. *Block diagram of the considered system.*

are then combined using MRC [12], [13], [17]–[21].

It is well known that the output SNR of maximum ratio combining is just the sum of the SNRs at the different receive antenna elements. For H-S/MRC, the instantaneous output SNR of H-S/MRC looks deceptively similar to MRC, namely

$$\gamma_{\text{H-S/MRC}} = \sum_{i=1}^{L} \gamma_{(i)} \,. \tag{2}$$

The big difference to MRC is that the $\gamma_{(i)}$ are the *ordered* SNRs, i.e., $\gamma_{(1)} > \gamma_{(2)} > \cdots > \gamma_{(N)}$. This leads to a different performance, and poses new mathematical challenges for the performance analysis. Specifically, we have to introduce the concept of "order statistics" [22]. Note that selection diversity (where only one out of N antennas is selected) and MRC are limiting cases of H-S/MRC with $L = 1$ and $L = N$, respectively.

In general, the gain of multiple antennas is due to two effects: "diversity gain" and "beamforming gain." The diversity gain is based on the fact that it is improbable that several antenna elements are in a fading dip simultaneously; the probability for very low SNRs is thus decreased by the use of multiple antenna elements. The "beamforming gain" is created by the fact that (with MRC) the combiner output SNR is the sum of the antenna SNRs. Thus, even if the SNRs at all antenna elements are identical, the combiner output SNR is larger, by a factor L, than the SNR at one antenna element. Antenna selection schemes provide good diversity gain, as they select the best antenna branches for combining. Actually, it can be shown that the diversity *order* obtained with antenna selection is proportional to N, not to L [23]. However, they do not provide full beamforming gain. If the signals at all antenna elements are completely correlated, then the SNR gain of H-S/MRC is only L, compared to N for an MRC scheme.

The analysis of H-S/MRC based on a chosen ordering of the branches at first appears to be complicated, since the SNR statistics of the ordered-branches are *not* independent. Even the *average* combiner output SNR calculation alone can require a lengthy derivation, as seen in [17]. However, we can alleviate this problem by transforming the ordered-branch variables into a new set of random variables. It is possible to find a transformation that leads to *independently distributed* random variables (termed "virtual branch variables") [12]. (When the average branch SNRs are not equal, it can be shown that the virtual branch variables are *conditionally* independent.) The fact that the combiner output SNR can be expressed in terms of i.i.d. virtual branch variables enormously simplifies the performance analysis of the system. For example, the derivation of the symbol error probability (SEP) for uncoded H-S/MRC systems, which normally would require the evaluation of nested N-fold integrals, essentially reduces to the evaluation of a *single* integral with finite limits.

The mean and the variance of the output SNR for H-S/MRC is thus [12]

$$\Gamma_{\text{H-S/MRC}} = L\left(1 + \sum_{n=L+1}^{N} \frac{1}{n}\right)\overline{\Gamma} \,, \tag{3}$$

and

$$\sigma^2_{\text{H-S/MRC}} = L\left(1 + L\sum_{n=L+1}^{N} \frac{1}{n^2}\right)\overline{\Gamma}^2 \,, \tag{4}$$

Channel estimation errors do not decrease the capacity significantly if the SNR of the pilot tones is comparable to, or larger than, the NR during the actual data transmission.

respectively, where $\overline{\Gamma}$ is the mean SNR.

The SEP for M-ary phase-shift keying (MPSK) with H-S/MRC is derived in [13] as

$$P^{\text{MPSK}}_{e,\text{H-S/MRC}} = \frac{1}{\pi}\int_0^{\Theta}\left[\frac{\sin^2\theta}{c_{\text{MPSK}}\overline{\Gamma} + \sin^2\theta}\right]^L \times \prod_{n=L+1}^{N}\left[\frac{\sin^2\theta}{c_{\text{MPSK}}\overline{\Gamma}\frac{L}{n} + \sin^2\theta}\right] d\theta, \tag{5}$$

where $\Theta = \pi(M-1)/M$, and $c_{\text{MPSK}} = \sin^2(\pi/M)$. Similar equations for the SEP for M-ary quadrature amplitude modulation (M-QAM) can be found in [13].

It is also important to note that the same principles can be used for MISO systems, i.e., where there are multiple antenna elements at the transmitter and only one antenna at the receiver. If the transmitter has complete channel state information (CSI), it can select transmit weights that are matched to the channel. If the transmitter uses all antenna elements, this is known as "maximum ratio transmission" (MRT) [24]; if antenna selection is applied, the system is called "hybrid—selection/maximum ratio transmission.

Performance of MIMO Systems

Diversity

As a next step, we analyze a diversity system that has multiple antenna elements both at the transmitter and at the receiver, and the transmitter has perfect CSI, i.e., know the matrix $\underline{H}$ completely). In the block diagram of Figure 1, our "space-time-coder" is then just a regular coder that puts out a sequence of scalar symbols s. These are then multiplied by the weight vector $\vec{u}$, to give the complex symbols at the different transmit antenna elements $\vec{s}$. Similarly, at the receiver, we obtain a "soft" symbol estimate r as $r = \vec{w}^* \vec{y}$. These symbols are then demodulated and decoded in the usual way (the "space-time decoder" is a conventional, scalar, decoder). In the following, we look at the case where the transmitter performs antenna selection, while the receiver uses all available signals and thus performs MRC. But the situation is reciprocal; all the following considerations are also valid if it is the receiver that performs the antenna selection.

The performance of this system was analyzed in [25], [26]. It is well known that any diversity system with CSI at the transmitter achieves an effective SNR that is equal to the square of the largest singular value of the channel matrix [27]. For a diversity system with antenna selection, we have to consider all possible antenna combinations. Each chosen set of antenna elements leads to a different channel matrix, and, thus, a different effective SNR. The antenna selection scheme finally chooses the matrix associated with the largest effective SNR.

In mathematical terms, that can be formulated the following way: define a set of matrices $\tilde{\underline{H}}$, where $\tilde{\underline{H}}$ is created by striking $N_t - L_t$ columns from $\underline{H}$, and $\mathcal{S}(\tilde{\underline{H}})$ denotes the set of all possible $\tilde{\underline{H}}$, whose cardinality is $\binom{N_t}{L_t}$. The achievable SNR γ of the reduced-complexity system (for a specific channel realization) is now

$$\gamma = \bar{\Gamma} \max_{\mathcal{S}(\tilde{\underline{H}})} \left(\max_i (\tilde{\lambda}_i^2) \right) \tag{6}$$

where the $\tilde{\lambda}_i$ are the singular values of $\tilde{\underline{H}}$. Analytical expressions for upper and lower bounds on the SNR, as well as Monte Carlo simulations of the exact results for the SNR and the bit error probability (BEP), and capacity derived from it are given in [25] and [26]. Note that the SNR of a diversity system is related to its capacity by the simple transformation $C = \log_2(1 + \gamma)$.

The *mean* SNR (averaged over all channel realizations) $E\{\gamma\}$ can be computed as [28]

$$E\{\gamma\} = \bar{\Gamma} \sum_{i=0}^{L-1} X_i \tag{7}$$

with

$$X_i = \frac{N_t!}{(i-1)!(N_t - i)!(N_r - 1)!} \sum_{r=0}^{i-1} (-1)^r \binom{i-1}{r} \times \sum_{s=0}^{(N_r-1)\xi} a_\xi \frac{\Gamma(1 + N_r + s)}{(\xi + 1)^{1+N_r+s}} \tag{8}$$

where ξ is $N_t - i + r$, and a_ξ is the coefficient of x^s in the expansion of $\sum_{l=0}^{N_r-1} (x^l / l!)^\xi$. The case $L_t = 1$ is analyzed in [29]–[31].

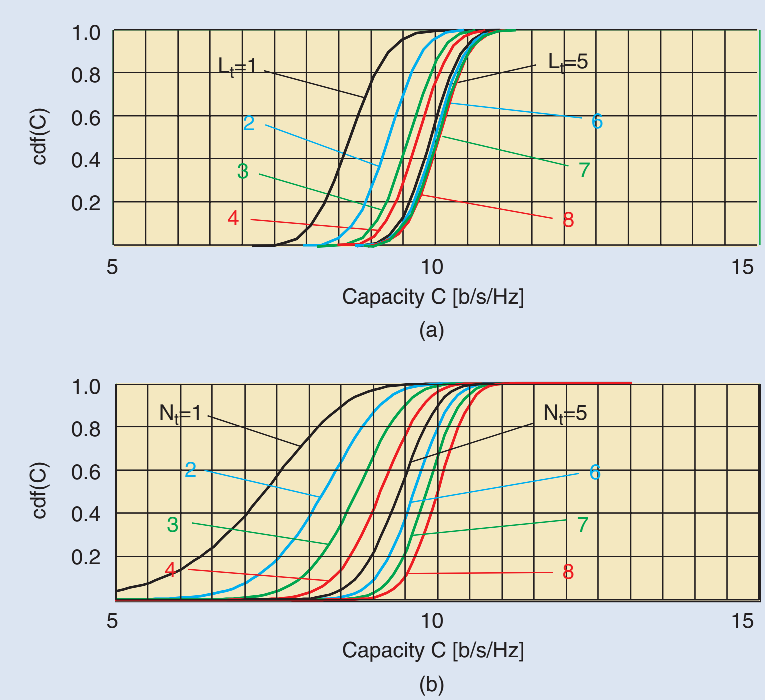

Figure 3. *(a) Capacity of a system with H-S/MRT at the transmitter and MRC at the receiver for various values of L_t with $N_t = 8$, $N_r = 2$, and SNR = 20 dB. (b) capacity of a system with MRT at transmitter and MRC at receiver for various values of N_t with $N_r = 2$, and SNR = 20 dB. From [26].*

Figure 3 shows the cdf of the capacity for H-S/MRT with different values of L_t, and compares it to MRT. We see that in this example (which uses $N_r = 2$, $N_t = 8$), the capacity obtained with $L_t = 3$ is already very close to the capacity of a full-complexity scheme. We also see that the improvement by going from one to three antennas is larger than that of going from three to eight. For comparison, we also show the capacity for pure MRT with different values of N_t. The required number of RF chains is L_t for the H-S/MRT case and N_t for the pure MRT case. Naturally, the capacity is the same for H-S/MRT with $L_t = 8$, and MRT with $N_t = 8$. It can be seen by comparing the two figures that, for a smaller number of RF chains, the H-S/MRT scheme is much more effective than a pure MRT scheme (for the same number of RF chains), both in terms of diversity order (slope of the curve) and ergodic capacity.

As discussed earlier, no diversity gain can be achieved by multiple antenna elements in correlated channels, and all gain is due to beamforming. Figure 4 compares the performance of a 3/8 H-S/MRT system with a 8/8 MRT. The outage capacity is plotted as a function of the ratio of the normalized correlation length at the transmitter (normalized to antenna spacing). As expected, the relative performance loss due to correlation is higher for the 3/8 H-S/MRT system than for the 8/8 MRT system. The mitigation of those problems by the introduction of a phase-shift only matrix that transforms the signals in the RF domain before selection takes place. has been suggested by [32] and [33]. This matrix can either be fixed, e.g., a fast Fourier transform (FFT), or adapting to the channel state. For fully correlated channels, this scheme can recover the beamforming gain. For i.i.d. channels, antenna selection with a fixed transformation matrix shows the same SNR distribution as a system without a transformation matrix; an adaptive transformation matrix, however, performs as well as a full-complexity system if $L \geq 2$.

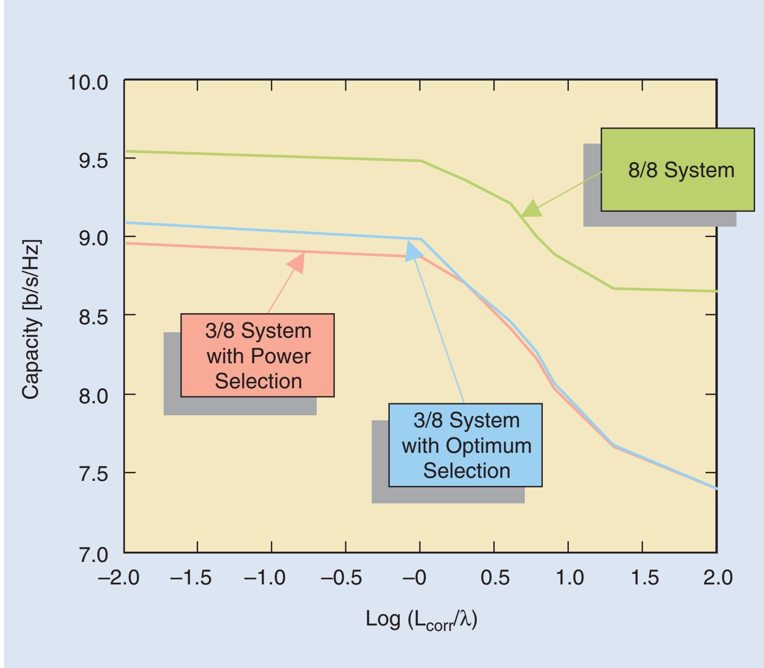

Figure 4. *A 10% outage capacity of a system with two receiving antennas and H-S/MRT at the transmitter as a function of the normalized correlation length at the transmitter (normalized to antenna spacing). A 3/8 system with optimum antenna selection (dashed), and 8/8 system (dotted). Correlation coefficient between signals at two antenna elements that are spaced d apart is* $\exp(-d/L_{\mathrm{corr}})$.

Spatial Multiplexing

For spatial multiplexing, different data streams are transmitted from the different antenna elements; in the following, we consider the case where the transmitter (TX), which has no channel knowledge, uses all antennas, while the receiver uses antenna selection [15]. In the block diagram of Figure 1, this means that the transmit switch is omitted. As we assume ideal (and unrestricted) processing in the space/time encoder/decoder, we do not need to consider the (linear) weights $\vec{u}$, $\vec{w}$ and can set them to unity.

Similar to the diversity case, each combination of antenna elements is associated with its own channel matrix $\underline{\tilde{H}}$. ($\underline{\tilde{H}}$ is created now striking $N_r - L_r$ rows from $\underline{H}$ because the selection occurs at the *receiver*.) However, the quantity we wish to optimize now is the information-theoretic capacity:

$$C_{\text{H-S/MIMO}} = \max_{S(\underline{\tilde{H}})} \left(\log_2 \left[\det \left(\underline{I}_{N_r} + \frac{\overline{\Gamma}}{N_t} \underline{\tilde{H}}\underline{\tilde{H}}^{\dagger} \right) \right] \right), \tag{9}$$

where $\underline{I}_{N_r}$ is the $N_r \times N_r$ identity matrix.

Let us first discuss from an intuitive point of view under what circumstances H-S/MIMO makes sense. It is immediately obvious that the number of parallel data streams we can transmit is upper-limited by the number of transmit antennas. On the other hand, we need at least as many receive antennas as there are data streams in order to separate the different data streams and allow demodulation. Thus, the capacity is linearly proportional to $\min(N_r, N_t)$ [3]. Any further increase of either N_r or N_t while keeping the other one fixed only increases the system diversity, and consequently allows a *logarithmic* increase of the capacity. But we have already seen in the previous section that hybrid antenna selection schemes provide good diversity. We can thus anticipate that a hybrid scheme with $N_r \geq L_r \geq N_t$ will give good performance.

An upper bound for the capacity for i.i.d. fading channels was derived in [15]. For $L_r \leq N_t$, this bound is

$$C_{\mathrm{H-S/MIMO}} \leq \sum_{i=1}^{L_r} \log_2 \left(1 + \frac{\overline{\Gamma}}{N_t} \gamma_{(i)} \right), \tag{10}$$

where the $\gamma_{(i)}$ are obtained by ordering a set of N_r i.i.d. chi-square random variables with $2N_t$ degrees of freedom (DOF). For $L_r > N_t$, the following bound is tighter

$$C_{\mathrm{H-S/MIMO}} \leq \sum_{j=1}^{N_t} \log_2 \left[1 + \frac{\overline{\Gamma}}{N_t} \sum_{i=1}^{L_r} \gamma_{(i)} \right] \tag{11}$$

where the $\gamma_{(i)}$ are obtained by ordering a set of N_r i.i.d. chi-square random variables with 2 DOF.

For the case that $L_t = N_r$, [34] derived a lower bound

$$C_{\mathrm{H-S/MIMO}} \geq \log_2 \left[\det \left(\underline{I}_{N_r} + \frac{\overline{\Gamma}}{N_t} \underline{\tilde{H}}\underline{\tilde{H}}^{\dagger} \right) \right] + \log_2 \left[\det \left(\underline{\tilde{U}}^{\dagger} \underline{\tilde{U}} \right) \right] \tag{12}$$

where $\underline{U}$ is an orthonormal basis of the column space of $\underline{H}$, and $\underline{\tilde{U}}$ is the $N_r \times L_t$ submatrix corresponding to the selected antennas. This equation can be used to derive further, looser but simpler bounds. The importance of this equation lies in the fact that the capacity loss $\log_2[\det(\underline{\widetilde{U}}^{\dagger}\underline{\widetilde{U}})]$ occurs as an additive term to the "usual" capacity expression, which has been investigated extensively.

Figure 5 shows the cdf of the capacity obtained by Monte Carlo simulations for $N_{\mathrm{r}} = 8$, $N_{\mathrm{t}} = 3$, and various L_{r}. With full exploitation of *all* available elements, a mean capacity of 23 b/s/Hz can be transmitted over the channel. This number decreases gradually as the number of selected elements L_{r} decreases, reaching 19 b/s/Hz at $L_{\mathrm{r}} = 3$. For $L_{\mathrm{r}} < N_{\mathrm{t}}$, the capacity decreases drastically, since a sufficient number of antennas to spatially multiplex N_{t} independent transmission channels is no longer available.

Correlation of the fading leads to a decrease in the achievable capacity (compare the decrease in diversity discussed earlier). One possibility for computing the performance loss is offered by (12): the performance loss of *any* MIMO system due to antenna selection is given by $\log_2[\det(\underline{\widetilde{U}}^{\dagger}\underline{\widetilde{U}})]$. This fact can be combined with well-known results for capacity of full-complexity MIMO systems in correlated channels [35] to give bounds of the capacity. The optimum transmit correlation matrix is derived in [36]. Phase transformation [32], [33], or beam selection [37] improve the performance in correlated channels. Also, the combination of constellation adaptation with subset selection is especially beneficial in correlated channels [38].

It also turns out that for antenna selection and low SNRs, diversity can give higher capacities than spatial multiplexing. This somewhat surprising result was proved in [39] . For small SNRs, the capacity with spatial multiplexing is

$$C_{\mathrm{H-S/MIMO}} \approx \frac{\gamma}{N_t \ln(2)} \sum_{i=1}^{L_{\mathrm{r}}} \sum_{j=1}^{L_{\mathrm{t}}} \left| \underline{\widetilde{H}}_{ij} \right|^2 \tag{13}$$

whereas for diversity, it is

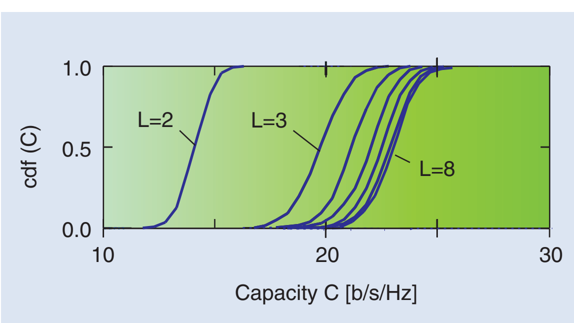

Figure 5. *Capacity for a spatial multiplexing system with $N_{\mathrm{r}} = 8$, $N_{\mathrm{t}} = 3$, SNR = 20 dB, and $L_{\mathrm{r}} = 2, 3, \ldots, 8$.*

$$C_{\mathrm{H-S/MRC}} \approx \frac{\gamma}{N_t \ln(2)} \sum_{i=1}^{L_{\mathrm{r}}} \left| \sum_{j=1}^{L_{\mathrm{t}}} \underline{\widetilde{H}}_{ij} \right|^2 . \tag{14}$$

In other words, the difference between the two expressions are the cross terms that appear for the diversity case. By appropriate choice of the antennas, the contribution from the cross terms to the capacity is positive, so that $C_{\mathrm{H-S/MRC}}$ can be larger than $C_{\mathrm{H-S/MIMO}}$. Similar results also hold in the case of strong interference [40].

Space-Time Coded Systems

Next, we consider space-time coded systems with transmit and receive antenna selection in correlated channels. We assume that the transmitter has knowledge about the *statistics* of the fading, i.e., it knows the correlation of the fading at the different antenna elements. Assume further that the so-called "Kronecker-model" is valid, in which the directions (and mean powers) of the multipath components at the transmitter are independent of those at the receiver [16], [41]. The channel with its selected antenna elements is then described by the modified correlation matrices $\underline{\widetilde{R}}_{\mathrm{t}}$ and $\underline{\widetilde{R}}_{\mathrm{r}}$, which describe the correlation of the signals at the selected antennas. The pairwise error probability (i.e., confusing codeword $\underline{S}^{(i)}$ with codeword $\underline{S}^{(j)}$) for a space-time coded system can then be shown to be [28], [42], [43]

$$P\left(\underline{S}^{(i)} \longrightarrow \underline{S}(j)\right) \leq \frac{\gamma^{-N_{\mathrm{t}}N_{\mathrm{r}}}}{|\underline{\widetilde{R}}_{\mathrm{t}}|^{N_{\mathrm{t}}} |\underline{\widetilde{R}}_{\mathrm{r}}|^{N_{\mathrm{r}}} |\underline{E}_{i,j}\underline{E}_{i,j}^{\dagger}|^{N_{\mathrm{r}}}} \tag{15}$$

where $\underline{E}_{i,j} = \underline{S}^{(i)} - \underline{S}^{(j)}$. The optimum antenna selection is thus the one that maximizes the determinants of $\underline{\widetilde{R}}_{\mathrm{t}}$ and $\underline{\widetilde{R}}_{\mathrm{r}}$. The selection at the transmitter and the receiver can be done independently; this is a consequence of the assumptions of the Kronecker model.

This equation also confirms that the achievable diversity order (which is the exponent of γ) is $N_{\mathrm{t}}N_{\mathrm{r}}$. However, note that the coding gain of a space-time coded antenna-selection system lies below that of a full-complexity system [44]. The combination of space-time block coding and antenna selection had also been suggested in [45]; specific results for the Alamouti code are given in [46]–[48]. Space-time trellis codes with antenna selection are analyzed in [49]. Code designs and performance bounds are given in [50].

Antenna Selection Algorithms

The only mechanism for a truly optimum selection of the antenna elements is an exhaustive search of all possible combinations for the one that gives the best SNR (for diversity) or capacity (for spatial multiplexing). However, this requires some $\binom{N_{\mathrm{t}}}{L_{\mathrm{t}}}\binom{N_{\mathrm{r}}}{L_{\mathrm{r}}}$ computations of determinants for each channel realizations, which quickly becomes impractical. For this reason, various simplified selection algorithms have been proposed. Most of them are intended for systems where the selection is done at only one link end.

The simplest selection algorithm is the one based on the power of the received signals. For the diversity case, this algorithm is quite effective. However, for spatial multiplexing, this approach breaks down. Only in about 50% of all channel realizations does the power-based selection give the same result as the capacity-based selection, and the resulting loss in capacity can be significant. This behavior can be interpreted physically: the goal of the receiver is to separate the different data streams. Thus, it is not good to use the signals from two antennas that are highly correlated, even if both have have high SNR. Figure 6 gives the capacities that are obtained by antenna selection based on the power criterion compared to the optimum selection.

Based on these considerations, an alternative class of algorithms has been suggested by [51]. Suppose there are two rows of the H that are identical. Since these two rows carry the same information, we can delete either of these two rows. In addition, if they have different powers (i.e., magnitude square of the norm of the row), we delete the row with the lower power. When there are no identical rows, we search for the two rows with highest correlation and then delete the row with the lower power. In this manner, we can have the channel matrix $\tilde{H}$ whose rows have minimum correlation and have maximum powers. This method achieves capacities within a few b/s/Hz. A somewhat similar approach, based on the mutual information either between receive antennas, or between transmit and receive antennas, has been suggested independently by [52].

Another algorithm was suggested in [53] (see also [54], [55]. It makes $N - L$ passes of a loop that eliminates the worst antenna, where the index $\tilde{p}$ of the worst antenna is found as

$$\tilde{p} = \arg\min_{p} \underline{H}_p \left[\underline{I} + \frac{E_s}{N_0} \underline{H}^\dagger \underline{H} \right]^{-1} \underline{H}_p^\dagger \tag{16}$$

where $\underline{H}_p$ is the pth row of $\underline{H}$. Further selection algorithms are also discussed in [56] and [57].

Effect of Nonidealities

Low-Rank Channels

Previously, we have assumed that the channel is i.i.d. complex Gaussian, or exhibits some correlation at the transmitter and/or receiver. However, in all of those cases, the channel matrix is full-rank, and the goal of the antenna selection is to decrease complexity, while keeping the performance loss as small as possible. There are, however, also propagation channels where the matrix $\underline{H}$ has reduced rank [58]–[60]. Under those circumstances, antenna selection at the transmitter can actually *increase* the capacity of the channel [61].

Note that the antenna selection increases the capacity only compared to the case of equal power allocation for all antennas. It cannot increase the capacity compared to the waterfilling approach; actually, the selection process can be considered as an approximation to waterfilling [62].

Linear Receivers for Spatial Multiplexing Systems

The simplest receiver for spatial multiplexing systems is a linear receiver that inverts the channel matrix $\underline{H}$ (zero-forcing). While this scheme is clearly suboptimal, it has the advantage of simplicity and is easy to analyze mathematically. The SNR for an M-stream spatial multiplexing system with a zero-forcing receiver was calculated in [63]. The SNR of the kth data stream γ_k is

$$\gamma_k^{(ZF)} = \frac{E_s}{MN_0 \left[\underline{\tilde{H}}_p^\dagger \underline{\tilde{H}}_p \right]_{kk}^{-1}} \tag{17}$$

which can be bounded as

$$\gamma_{\min}^{(ZF)} \geq \lambda_{\min}^2 \left(\underline{\tilde{H}} \right) \frac{E_s}{MN_0}. \tag{18}$$

Frequency-Selective Channel

In frequency-selective channels, the effectiveness of antenna selection is considerably reduced. Note that different sets of antenna elements are optimum for different (uncorrelated) frequency bands. Thus, in the limit that the system bandwidth is much larger than the

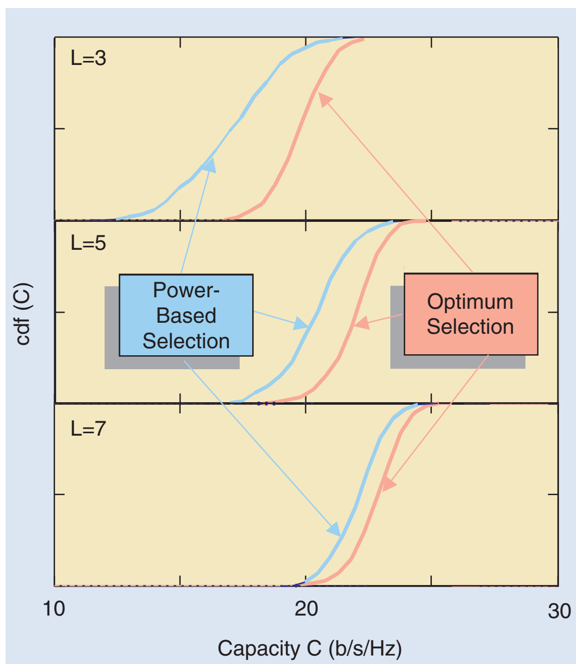

Figure 6. *Cdf of the capacity of a system with $N_r = 8$, $N_t = 3$. Selection of antenna by capacity criterion (solid) and by power criterion (dashed).*

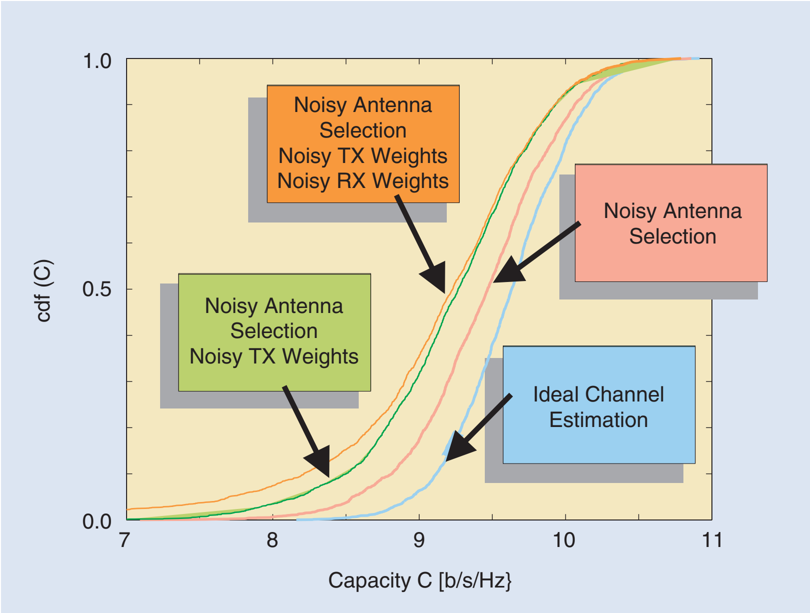

Figure 7. *Impact of errors in the estimation of transfer function matrix H. Cdf of the capacity for 1) ideal CSI at TX and RX (solid), 2) imperfect antenna selection, but perfect antenna weights (dashed), 3) imperfect antenna section as well as weights at TX only (dotted), and 4) imperfect antenna weights at TX and RX (dash-dotted).* $SNR_{\text{pilot}} = 5$ *dB. From [65].*

coherence bandwidth of the channel, and if the number of resolvable multipath components is large, all possible antenna subsets become equivalent. This can also be interpreted by the fact that such a system has a very high diversity degree, so that any additional diversity from antenna selection would be ineffective anyway. However, for moderately frequency-selective channels, antenna selection still gives significant benefits. A precoding scheme for code division multiple access (CDMA) that achieves such benefits is described in [64].

Channel Estimation Errors

We next investigate the influence of erroneous CSI on a diversity system with transmit antenna selection [65]. We assume that in a first stage, the complete channel transfer matrix is estimated. Based on that estimate, the antennas that are used for the actual data transmission are selected, and the antenna weights are determined. Erroneous CSI can manifest itself in different forms, depending on the configuration of the training sequence and the channel statistics: 1) erroneous choice of the used antenna elements, 2) errors in the transmit weights, and 3) errors in the receive weights. Figure 7 shows the effect of those errors on the capacity of a diversity system. The errors in the transfer functions are assumed to have a complex Gaussian distribution with certain SNR_{pilot}, which is the SNR during the transmission of the pilot tones. We found that for an SNR_{pilot} of 10 dB results in a still tolerable loss of capacity (less than 5%). However, below that level, the capacity starts to decrease significantly, as depicted in Figure 7.

Another type of channel estimation error can be caused by a limited feedback bit rate (for feeding back CSI from the receiver to the transmitter in a frequency-duplex system). This problem is especially important for the W-CDMA standard, where the number of feedback bits is limited to two per slot. ßAttempts to send the weight information for many transmit antennas, thus, have to be in a very coarse quantization, or have to be sent over many slots, so that—in a time-variant environment—the feedback information might be outdated by the time it arrives at the transmitter. Thus, the attempt of getting full CSI to the transmitter carries a penalty of its own. The use of hybrid antenna selection might give better results in this case, since it reduces the number of transmit antennas for which channel information has to be fed back. An algorithm for optimizing the "effective" SNR is discussed in [66].

Hardware Aspects

Finally, we consider the effects of the hardware on the performance. In all the previous sections, we had assumed "ideal" RF switches with the following properties:

- They do not suffer any attenuation or cause additional noise in the receiver.
- They are capable of switching instantaneously.
- They have the same transfer function irrespective of the output and input port.

Obviously, those conditions cannot be completely fulfilled in practice:

- The attenuation of typical switches varies between a few tenths of a decibel and several decibels, depending on the size of the switch, the required throughput power (which makes TX switches more difficult to build than RX switches), and the switching speed. In the TX, the attenuation of the switch must be compensated by using a power amplifier with higher output power. At the receiver, the attenuation of the switch plays a minor role if the switch is placed *after* the low-noise receiver amplifier (LNA). However, that implies that N_r instead of L_r receive amplifiers are required, eliminating a considerable part of the hardware savings of antenna selection systems.
- Switching times are usually only a minor issue. The switch has to be able to switch between the training sequence and the actual transmission of

the data, without decreasing the spectral efficiency significantly. In other words, as long as the switching time is significantly smaller than the duration of the training sequence, it does not have a detrimental effect.

- The transfer function has to be the same from each input-port to each output-port, because otherwise the transfer function of the switch distorts the equivalent baseband channel transfer function that forms the basis of all the algorithms. It cannot be considered part of the training because it is not assured that the switch uses the same input-output path during the training as it does during the actual data transmission. An upper bound for the admissible switching errors is the error due to imperfect channel estimation.

Summary and Conclusions

This article presented an overview of MIMO systems with antenna selection. Either the transmitter, the receiver, or both use only the signals from a subset of the available antennas. This allows considerable reductions in the hardware expense. The most important conclusions are

- Antenna selection retains the diversity degree, compared to the full-complexity system, for both linear diversity systems with complete channel knowledge and space-time coded systems. However, there is a penalty in terms of the average SNR.
- For spatial multiplexing systems (BLAST), antenna selection at the receiver only gives a capacity comparable to the full-complexity system as long as $L_r \geq N_t$ (and similarly for the selection at the transmitter).
- Optimum selection algorithms have a complexity $\binom{N}{L}$. However, fast selection algorithms do exist that have much lower (polynomial with N) complexity, and perform almost as well as full-complexity systems.
- For low SNR, spatial multiplexing does not necessarily maximize capacity when antenna selection is present. The same is true for strong interference.
- For low-rank channels, transmit antenna selection can *increase* the capacity compared to a full-complexity system without channel knowledge at the TX.
- Channel estimation errors do not decrease the capacity significantly if the SNR of the pilot tones is comparable to, or larger than, the SNR during the actual data transmission.
- Frequency selectivity reduces the effectiveness of antenna selection.
- Switches with low attenuation are required both for transmitter and receiver.
- Antenna selection is an extremely attractive scheme for reducing the hardware complexity in MIMO systems.

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