Resource Management Games for Distributed Network Localization

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***Abstract*— Resource management in the power and time–frequency domains is an important issue in distributed network localization. Since highly accurate ranging requires a large amount of time–frequency resources, cooperation among nodes without proper link selection may not be feasible. To address this issue, two resource management games are formulated, and Stackelberg equilibrium and link bargaining equilibrium are proposed as the solution concepts for efficient link selection and power allocation. Distributed algorithms are derived and analyzed using game theoretical approaches. It is demonstrated that the proposed strategies can achieve a lower mean squared error of position estimation with fewer ranging measurements.**

***Index Terms*— Cooperative localization, game theory, link selection, Nash equilibrium, power allocation.**

I. Introduction

HIGH-ACCURACY LOCALIZATION has become an essential component in many emerging applications, such as indoor navigation and logistic automation [1]–[12]. However, in harsh propagation environments, high-accuracy localization is challenging [13]–[22]. For example, in indoor scenarios, the global positioning system (GPS) fails to provide accurate position estimates due to poor reception of signals from GPS satellites. In addition, received signal strength techniques may not provide satisfactory position accuracy.

To improve the position accuracy in harsh propagation environments, cooperative localization techniques are proposed in [13] and [15], where mobile agents with imperfect position knowledge share their position information and take range measurements between each other. In addition, power allocation over range measurements on different links was proposed in [8] and [17], with the objective to minimize the network square position error bound (SPEB) subject to total power constraints. However, these studies only improve the efficiency of network localization in the power domain. In practice, it is still challenging to apply agent network localization techniques due to the following issues.

First, high-accuracy cooperative localization requires a large amount of time-frequency resources. Consider that the agents are densely deployed, and hence are fully connected with each other. The number of candidate links for range measurements is $K(K-1)/2$, where K is the number of agents. In particular, for an asynchronous network, where round-trip time-of-arrival (TOA) ranging is required, the number of range measurements in the network scales as $K(K-1)$. Moreover, high-accuracy ranging requires wideband signals, which cannot be multiplexed in the frequency domain. Furthermore, as ranging signals are usually transmitted using distributed multiple access protocols, additional overhead in the time domain should be taken into account due to potential signal collisions. However, existing power allocation algorithms do not constrain the total number of links used for range measurements, and hence are not scalable when the number of agents K grows.

Second, since agent networks are usually formed in an ad-hoc manner, the existing distributed power allocation algorithms in [15] and [23] require large overhead for message passing among agents, which is not affordable when the number of agents becomes large. Therefore, an efficient coordination mechanism for distributed link selection and power allocation among the agents is needed.

Third, although there are some studies using game theory as a tool to design distributed algorithms in cooperative localization [24]–[29], those studies focused on localizing target nodes through a set of anchors operating in a distributed manner. Broadly, game theoretical approaches have been applied to many scenarios in communication networks, such as power control in CDMA systems [30]–[33], dynamic spectrum access and resource allocation in cognitive radio networks [34]–[36], and precoding strategies for interference channels [37]–[39]. Zero-sum differential games for double-sided jamming were studied in [40]–[42], where hierarchical solutions were derived. In cooperative localization, coalitional game approaches were used to develop algorithms for sleep time allocation among anchor nodes [25], [26], dynamic range

Manuscript received April 30, 2016; revised September 30, 2016; accepted November 28, 2016. Date of publication January 26, 2017; date of current version March 31, 2017. This work was supported in part by the Air Force Office of Scientific Research under Grant FA9550-12-0287, in part by the MIT Institute for Soldier Nanotechnologies, and in part by the National Natural Science Foundation of China under Grant 61501279.

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Digital Object Identifier 10.1109/JSAC.2017.2659318

measurement allocation [27], and node selection [28] or link selection [29] in forming a cooperative network of anchors. However, there are few results for game theoretical approaches to link selection and power allocation in cooperative self-localization for agents. Prior studies [23] and [43] used game theory for power management in cooperative localization, where agents do not have link selection constraints on taking range measurements.

This paper aims to address the above issues from two aspects. First, we reduce the number of range measurements from $\mathcal{O}(K^2)$ as in prior studies to $\mathcal{O}(K)$, hence increasing the time-frequency efficiency for distributed network localization. The corresponding challenge is to determine the link selection in a distributive way. Second, we reduce the coordination overhead for power allocation over the selected links. The main contributions of this paper are summarized as follows:

- We propose a resource management game to determine the link selection and power allocation for distributed network localization. Numerical results show that a lower mean squared error (MSE) of position estimation is achieved with fewer range measurements.
- We develop a distributed link selection algorithm that requires only local information, such as the channel qualities of the agents.
- When the network degenerates into tree subnetworks after the link selection, we exploit the hierarchical information structure and adopt the Stackelberg equilibrium (SE) as the solution concept for efficient power allocation strategies.
- When the subnetworks have general topologies, we propose a new solution concept as link bargaining equilibrium (LBE) that is obtained by an algorithm based on per link negotiation with only a small amount of coordination.

The rest of the paper is organized as follows. Section II illustrates the distributed network localization model, resource management game formulation, and the general solution concepts. Section III studies the tree topology network and the Stackelberg game that exploits the hierarchical information structure. Section IV studies the LBE obtained from per link negotiation for a general network. Numerical results are demonstrated in Section V, and conclusions are given in Section VI.

Notations: Bold characters $\mathbf{a}$ and $\mathbf{A}$ denote a vector and a matrix, respectively. The notations $\mathbf{a} \succeq \mathbf{b}$ means $a_i \geq b_i$ for each i, and $\mathbf{A} \succeq \mathbf{0}$ means $\mathbf{A}$ is a positive semidefinite (PSD) matrix.

II. System Model

This section illustrates the distributed network localization model and reviews the existing power management games.

A. Cooperative Localization

Consider a network with K agents, given by a set $\mathcal{K} = \{1, 2, \ldots, K\}$. Each agent $k \in \mathcal{K}$ has an initial estimation $\hat{\mathbf{p}}_k^{\mathrm{o}}$ of its position $\mathbf{p}_k \in \mathbb{R}^2$ from the anchors, and the associated accuracy is captured by a 2×2 equivalent EFIM [13], denoted

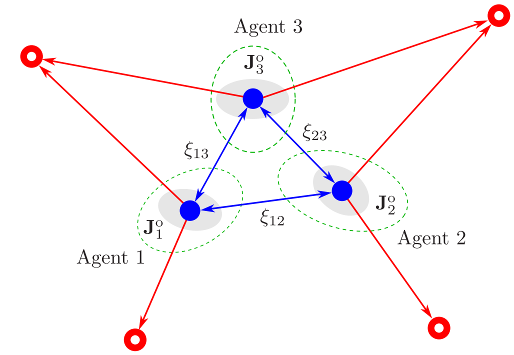

Fig. 1. Illustration of the cooperative localization: agents (blue dots) obtain initial position estimates from anchors (red circles), and the grey ellipses represent the initial EFIM [13]. The agents communicate with neighboring agents to improve position accuracy, and the green ellipses represent the expanded EFIM after cooperation to infer their positions.

as $\mathbf{J}_k^{\mathrm{o}}$. Consider that agents are not synchronized. To increase the position accuracy, agent k first selects a set of neighbors $\mathcal{N}(k)$, and then each pair of agents $\{(k, j) : j \in \mathcal{N}(k)\}$ perform two-way TOA range measurements. Finally, new position estimations $\hat{\mathbf{p}}_k$ are obtained based on $\hat{\mathbf{p}}_k^{\mathrm{o}}$, $\mathbf{J}_k^{\mathrm{o}}$, and the results of the range measurements. Note that we focus on the performance after the agents have undergone one round of cooperation. Fig. 1 illustrates cooperative localization with three agents.

It has been shown in [13] that the MSE of the position estimation for agent $k \in \mathcal{K}$ is bounded by the following individual SPEB as

$$\mathbb{E}\left\{\|\hat{\mathbf{p}}_k - \mathbf{p}_k\|^2\right\} \geq \mathrm{tr}\{\mathbf{J}_k^{-1}\} \tag{1}$$

where $\mathbf{J}_k$ is the 2×2 individual EFIM for agent k after cooperation. To specify $\mathbf{J}_k$, let x_{kj} be the transmit power sent from agent k to agent j, and let ξ_{kj} be the channel quality [8] between agent k and agent j. The individual EFIM $\mathbf{J}_k$ can be determined through the following result.

Lemma 1 (Individual EFIM [43]): For round-trip TOA ranging in an asynchronous network,[1] the individual EFIM $\mathbf{J}_k$ for agent k can be expressed as

$$\mathbf{J}_k = \mathbf{J}_k^{\mathrm{o}} + \sum_{l \in \mathcal{N}(k)} g_{kl}(x_{kl}, x_{lk}) \mathbf{u}_{kl} \mathbf{u}_{kl}^{\mathrm{T}} \tag{2}$$

where

$$g_{kl}(x_{kl}, x_{lk}) \triangleq \frac{4 x_{kl} x_{lk} \xi_{kl}}{x_{kl} + x_{lk} + 4 x_{kl} x_{lk} \xi_{kl} \delta_{kl}} \tag{3}$$

in which

$$\delta_{kl} = \mathbf{u}_{kl}^{\mathrm{T}} (\mathbf{J}_l^{\mathrm{o}})^{-1} \mathbf{u}_{kl} \tag{4}$$

the vector $\mathbf{u}_{kj} = [\cos\phi_{kj} \ \ \sin\phi_{kj}]^{\mathrm{T}}$ captures the ranging direction, and ϕ_{kj} denotes the angle between agent k and j.

In addition, we assume the agents have only local information of their neighbors: each agent k knows the information of the agents j that connect to it, e.g., the channel quality ξ_{kj},

[1] Throughout this paper, the analysis and insights mainly focus on asynchronous networks. The results for synchronous networks are similar, with details omitted due to page limitation.

the initial estimation $\hat{\mathbf{p}}_j^{\text{o}}$, and $\mathbf{J}_j^{\text{o}}$; however, agent k does not know the information between agent l and m, e.g., the channel quality ξ_{lm}.

B. Problem Formulation

Let $\mathbf{x}_k \triangleq \{x_{kj}\}_{j\neq k}$ be the collection of power allocation variables of agent k, and let $\mathbf{x}_{-k} \triangleq \{\mathbf{x}_j\}_{j\neq k}$ be the power allocation variables of all the other agents. Each agent k has its own objective (cost function) to minimize the individual SPEB[2] penalized by the power consumption, formulated as

$$f_k(\mathbf{x}_k, \mathbf{x}_{-k}) \triangleq \text{tr}\{\mathbf{J}_k(\mathbf{x}_k, \mathbf{x}_{-k})^{-1}\} + V_k \sum_{j\in\mathcal{N}(k)} x_{kj} \tag{5}$$

where $V_k > 0$ is an agent-specific power-conservative level, and the term $V_k \sum_{j\in\mathcal{N}(k)} x_{kj}$ characterizes the power cost.

Three types of constraints are imposed. First, each agent k has a power budget $P^{(k)}$ for the range measurement

$$x_{kj} \geq 0, \quad \sum_{j\in\mathcal{N}(k)} x_{kj} \leq P^{(k)}, \qquad \forall k \in \mathcal{K}. \tag{6}$$

For notational convenience, we define the set of feasible power allocations for agent k as $\mathcal{P}_k \triangleq \{\mathbf{x}_k : x_{kj} \geq 0, \sum_{j\in\mathcal{N}(k)} x_{kj} \leq P^{(k)}\}$.

Second, consider a decomposition of the variable x_{kj} into a link selection variable $\eta_{kj} \in \{0, 1\}$ and a power allocation variable $\widetilde{x}_{kj}$

$$x_{kj} = \eta_{kj}\widetilde{x}_{kj}, \qquad \forall k, j. \tag{7}$$

There are link selection constraints to limit the number of range measurements taken in the network:

$$\eta_{kj} = \eta_{jk} \in \{0, 1\}, \quad \forall k \neq j \tag{8}$$

$$\frac{1}{2}\sum_{k\in\mathcal{K}}\sum_{j\neq k} \eta_{kj} \leq K\bar{L} \tag{9}$$

where $\bar{L}$ is a system parameter to be specified and constraint (9) restricts the total number of links to be no larger than $K\bar{L}$, which means there are at most $2K\bar{L}$ two-way TOA range measurements taken in the network. This scheme is more easily scalable to a large network as compared to the traditional case, where almost all links are selected for cooperation (i.e., the number of range measurements scales as $\mathcal{O}(K^2)$).

Third, we consider a fairness constraint on the link selection

$$\sum_{j\neq k} \eta_{kj} \geq \bar{L}, \quad \forall k \in \mathcal{K} \tag{10}$$

which assigns at least $\bar{L}$ links to each agent k. This is to avoid the scenario where some agents may attract most of the potential links for range measurements,[3] resulting in few or no links available for the other agents due to the total link constraint (9).

[2]The individual SPEB in (1) is used as the performance metric for position accuracy because it is tight in high signal-to-noise ratio (SNR) regimes, as demonstrated by the numerical results in Section V.

[3]For example, the agents with better prior information $\hat{\mathbf{J}}_k^{\text{o}}$ may attract more cooperation from other agents due to (4).

The resource management game $\mathcal{G}$ can be written as a three-tuple $(\mathcal{K}, \prod_k \mathcal{X}_k, \mathbf{f})$, where

$$\mathcal{X}_k(\mathbf{x}_{-k}) = \{x_{kj} : \forall j \in \mathcal{N}(k), (6) - (10) \text{ are satisfied}\}$$

is the strategy set to specify the feasible link selection and power allocation, in which the variable $\mathbf{x}_{-k}$ emphasizes that the strategy set of agent k depends on the strategies of the other agents, and $\mathbf{f} = (f_1, f_2, \ldots, f_K)$ is the cost function vectors of all the agents.

A straight-forward solution concept to game $\mathcal{G}$ is the Nash equilibrium (NE).

Definition 1 (Nash Equilibrium): A solution profile $\mathbf{x}^* = (\mathbf{x}_1^*, \mathbf{x}_2^*, \ldots, \mathbf{x}_K^*)$ is called an NE (in pure strategies) if and only if the following holds for each agent k:

$$f_k(\mathbf{x}_k^*, \mathbf{x}_{-k}^*) \leq f_k(\mathbf{x}_k, \mathbf{x}_{-k}^*) \tag{11}$$

for all $\mathbf{x}_k \in \mathcal{X}_k(\mathbf{x}_{-k}^*)$.

Since zero power allocation $\mathbf{x} = \mathbf{0}$ is always an NE, the concept of a *cooperating NE* is introduced as follows.

Definition 2 (Cooperating NE): A cooperating NE is an NE $\mathbf{x}^*$ that has at least one non-zero component.

Note that since the link selection constraints (8)–(10) couple all the agents, computing the NE usually involves global information exchange or centralized computing. Meanwhile, since the agents may change their locations, it is not feasible to implement link selection and power allocation algorithms that require a large amount of communication overhead and take a long time to converge. Therefore, instead of pursuing the NE solution, we study two different solution concepts in Section III and IV, and the NE is used to provide a performance benchmark.

III. Game in a Tree Topology Under $\bar{L} = 1$

This section investigates the scenario of $\bar{L} = 1$ in the link selection constraints (9) and (10). The motivations are twofold. First, this scenario corresponds to a conservative case where there are at most $2K$ range measurements in the network. Second, under this scenario, a tree topology can be formed for the agent network, and a hierarchical structure can be exploited for efficient distributed power allocation algorithms.

A. Distributed Link Selection

Consider a simple strategy where two agents on link (k, j) perform power allocation by ignoring all the other links. This corresponds to the worst-case situation, where all the other agents do not want to cooperate with agents k and j.

Specifically, define the link individual cost function for agent k as

$$f_{kj}(x_{kj}, \mathbf{x}_{-k}) \triangleq \text{tr}\left\{\left[\mathbf{J}_k^{\text{o}} + g_{kj}(x_{kj}, x_{jk})\mathbf{u}_{kj}\mathbf{u}_{kj}^{\text{T}}\right]^{-1}\right\} + V_k x_{kj}. \tag{12}$$

Note that $f_{kj}(x_{kj}, \mathbf{x}_{-k}) = f_k(\mathbf{x}_k, \mathbf{x}_{-k})$ for $\mathbf{x}_k = (0, 0, \ldots, x_{kj}, \ldots, 0)$; i.e., non-zero power allocation only applies to link (k, j) for agent k. Consider that agents k and j

choose the power allocation as the solution to a per link negotiation subgame formulated as follows:

$$\underset{x_{kj}\geq 0}{\text{minimize}}\ f_{kj}(x_{kj}, \mathbf{x}_{-k}) \tag{13}$$

$$\underset{x_{jk}\geq 0}{\text{minimize}}\ f_{jk}(x_{jk}, \mathbf{x}_{-j}) \tag{14}$$

where the NE of the subgame is a solution profile $\{(x_{kj}^*, x_{jk}^*)\}$ that satisfies $f_{kj}(x_{kj}^*, \mathbf{x}_{-k}^*) \leq f_{kj}(x_{kj}, \mathbf{x}_{-k}^*)$, for all $x_{kj} \geq 0$, and $f_{jk}(x_{jk}^*, \mathbf{x}_{-j}) \leq f_{jk}(x_{jk}, \mathbf{x}_{-j})$, for all $x_{jk} \geq 0$.

Even though there is only one candidate link (k, j) in the per link negotiation subgame (13)–(14), deriving the NE solution is non-trivial. Instead, we derive the condition of non-zero solution as follows.

Lemma 2 (Condition for Cooperation): The per link negotiation subgame (13)-(14) for agents k and j admits a cooperating NE $x_{kj}^*, x_{jk}^* \neq 0$ if and only if

$$\xi_{kj} > \frac{1}{4}\left(\sqrt{\Gamma_{kj}} + \sqrt{\Gamma_{jk}}\right)^2 \tag{15}$$

where

$$\Gamma_{kj} \triangleq \frac{V_k}{\mathbf{u}_{kj}^{\mathrm{T}}(\mathbf{J}_k^{\mathrm{o}})^{-2}\mathbf{u}_{kj}} \tag{16}$$

is defined as the conservative coefficient of agent k on link (k, j). Moreover, the cooperating NE is unique.

Proof: The per link negotiation subgame corresponds to the two-agent power management game in [43]. Hence, the result is a direct application of [43, Th. 1]. □

Following the result in Lemma 2, define the cooperation quality over link (k, j) as

$$\Xi_{kj} \triangleq 4\xi_{kj}\left(\sqrt{\Gamma_{kj}} + \sqrt{\Gamma_{jk}}\right)^{-2}. \tag{17}$$

Note that if $\Xi_{kj} > 1$, then a cooperating NE with non-zero power allocation is obtained for a two-agent subnetwork. Intuitively, if $\Xi_{kj} \ll 1$, the power allocation from the per link negotiation subgame may yield $x_{kj}^* = x_{jk}^* = 0$, which implies that it may not be a good choice to select link (k, j) for cooperation.

Following this insight, the links with high cooperation quality Ξ_{kj} should be selected with high priority. Specifically, to satisfy constraints (8)–(10), a distributed sub-optimal link selection algorithm can perform as follows. First, each agent selects $\bar{L}$ links that correspond to the $\bar{L}$-highest cooperation quality Ξ_{kj}. Then, the cooperation network is formed by all the agents and links that are selected by any one of the agents. For example, if link (k, j) is selected by agent k and link (l, k) is selected by agent l, then agent k has at least two links (k, j) and (l, k) for cooperation. The link selection algorithm is summarized in Algorithm 1. Note that Algorithm 1 only requires local information to be available for each agent.

Algorithm 1 Link Selection for General $\bar{L} \geq 1$

1) Each agent k selects $\bar{L}$ links that correspond to the $\bar{L}$-highest cooperation quality Ξ_{kj} in (17).
2) A partially connected agent network is formed, where the nodes are the agents and the edges are the links selected by any one of the agents.

B. Stackelberg Game for Power Allocation

1) Tree Topology: It turns out that under $\bar{L} = 1$, Algorithm 1 yields a tree topology. To see this, some concepts are introduced as follows.

A graph in a cooperative localization network is given by a set of nodes and edges, where the nodes represent the agents and the edges represent the wireless links connecting the agents. The degree of the node is the number of edges that connect to the node. A graph is connected if there is a path between every pair of nodes. A (rooted) tree is a connected graph of K nodes and $K - 1$ edges, with one node designated as the root. A leaf node in a tree is a node with a degree of 1, apart from the root. Moreover, we have the following definition of the internal node.

Definition 3 (Internal Node): An internal node is either a node with a degree of at least 2 or the root.

Proposition 1 (Tree Subnetworks): Under $\bar{L} = 1$, Algorithm 1 decomposes the network into a number of subnetworks with a tree topology.

Proof: After link selection, the original fully connected graph is decomposed into one or more connected subgraphs. Each subgraph that has K_1 nodes has exactly $K_1 - 1$ edges. This is because, on the one hand, if the subgraph has fewer than $K_1 - 1$ edges it is not connected, and on the other hand, the subgraph cannot have more than $K_1 - 1$ edges since the K_1 nodes can select at most $\bar{L}K_1$ distinct edges. However, there must be one edge that has the largest link cooperation quality within the subgraph and it must be selected by both of the nodes at the same time. Thus, there are at most $K_1 - 1$ distinct edges selected. Hence, the subgraph is a tree. □

As a result of Proposition 1, the link constraint in (9) is satisfied.

2) Hierarchical Information Structure and the Leader-Follower Model: As internal nodes have more links than leaf nodes (single link), the tree topology yields a hierarchical information structure in the network. To elaborate this observation, consider that the best-response strategy [43] for power allocation is followed by all the agents. In particular, the internal node k can calculate the best-response strategy of the leaf node j as follows:

$$x_{jk}^{\mathrm{BR}} = T_{jk}(x_{kj}) \triangleq \arg\min_{\mathbf{x}_j \in \mathcal{P}_j} f_j(\mathbf{x}_j, \mathbf{x}_{-j}) \tag{18}$$

where the power allocation variable $\mathbf{x}_j = \{x_{jk}\}$ for leaf node j and the variable $\mathbf{x}_{-j} = \{x_{kj}\}$ from node j's single neighbor (node k) is a scalar due to the network topology. In turn, leaf node j cannot specify the best-response strategy of internal node k because node j does not know the information between internal node k and k's other neighbors besides itself.

With the hierarchical information structure, a leader-follower model can be used to determine the power allocation. The internal nodes act as leaders, and the leaf nodes act as followers. The leaders can determine the power allocation as being aware of the best-response strategies played by the followers. Such a hierarchical leader-follower game can be formulated and analyzed under a Stackelberg game framework as follows.

3) Stackelberg Game Formulation: Consider a tree subnetwork. Let $\mathcal{K}_{\text{int}}$ be the set of internal nodes of the tree, $\mathcal{N}_{\text{le}}(k)$ be the set of leaf neighbor nodes of node k, and $\mathcal{N}_{\text{int}}(k) = \mathcal{N}(k) \backslash \mathcal{N}_{\text{le}}(k)$ be the set of internal neighbor nodes of node k. As a result, $\mathcal{K}_{\text{int}}$ and $\mathcal{N}_{\text{int}}(k)$ are sets of leader agents, and $\mathcal{N}_{\text{le}}(k)$ are sets of follower agents. Using (2), (5), and (18), the Stackelberg power allocation game is formulated as follows:

$$\underset{\mathbf{x}_k \in \mathcal{P}_k}{\text{minimize}} \; f_k^{\text{c}}(\mathbf{x}_k, \mathbf{x}_{-k}), \quad \forall k \in \mathcal{K}_{\text{int}} \tag{19}$$

$$\underset{\mathbf{x}_j \in \mathcal{P}_j}{\text{minimize}} \; f_j(\mathbf{x}_j, \mathbf{x}_{-j}), \quad \forall j \in \mathcal{K} \backslash \mathcal{K}_{\text{int}} \tag{20}$$

where

$$f_k^{\text{c}}(\mathbf{x}_k, \mathbf{x}_{-k}) = \text{tr}\Big\{ \Big[\mathbf{J}_k^{\text{o}} + \sum_{l \in \mathcal{N}_{\text{le}}(k)} g_{kl}(x_{kl}, T_{lk}(x_{kl})) \mathbf{u}_{kl} \mathbf{u}_{kl}^{\text{T}} + \sum_{j \in \mathcal{N}_{\text{int}}(k)} g_{kj}(x_{kj}, x_{jk}) \mathbf{u}_{kj} \mathbf{u}_{kj}^{\text{T}} \Big]^{-1} \Big\} + V \sum_{j \in \mathcal{N}(k)} x_{kj} \tag{21}$$

is the modified objective function for a leader agent, which is aware of its followers' strategies, and $T_{lk}(\cdot)$ is the best-response power allocation function defined in (18) for a follower agent $l \in \mathcal{N}_{\text{le}}(k)$ of the leader agent k.

The merit of the Stackelberg game (19)–(20) is that it only requires iterations among the leader agents.

Consider the link selection from Algorithm 1 followed by the Stackelberg power allocation (19)-(20). The solution concept of SE is defined as follows.

Definition 4 (Stackelberg Equilibrium (SE)): Given an agent partition $\{\mathcal{K}_{\text{int}}, \mathcal{K} \backslash \mathcal{K}_{\text{int}}\}$ and a network topology, a power allocation profile $\mathbf{x}^* \neq \mathbf{0}$ is called an SE if and only if the following holds:

$$f_k^{\text{c}}(\mathbf{x}_k^*, \mathbf{x}_{-k}^*) \leq f_k^{\text{c}}(\mathbf{x}_k, \mathbf{x}_{-k}^*), \quad \forall \mathbf{x}_k \in \mathcal{P}_k$$

for $k \in \mathcal{K}_{\text{int}}$,

$$f_j(\mathbf{x}_j^*, \mathbf{x}_{-j}^*) \leq f_j(\mathbf{x}_j, \mathbf{x}_{-j}^*), \quad \forall \mathbf{x}_j \in \mathcal{P}_j$$

for $j \in \mathcal{K} \backslash \mathcal{K}_{\text{int}}$.

C. Performance Advantage

We first show the convexity of the modified objective function (21).

Proposition 2 (Convexity of f_k^c): The function $f_k^{\text{c}}(\mathbf{x}_k, \mathbf{x}_{-k})$ is convex in $\mathbf{x}_k$.

Proof: Please refer to Appendix A for the proof. □

Since $f_k^{\text{c}}(\mathbf{x}_k, \mathbf{x}_{-k})$ is convex, there is a unique local minimizer $\mathbf{x}_k^*$, which can be efficiently found by various numerical methods. Therefore, implementing the modified strategy in the internal node does not increase the computational complexity by much, and hence the internal nodes do not sacrifice themselves in computational complexity by being leaders.

Consider a two-agent network. without loss of generality (w.l.o.g.), we assign agent 1 to be the leader and agent 2 to be the follower. In general Stackelberg games, the leaders can be shown to have a performance advantage, but the advantage for the followers is not guaranteed. However, it is shown in the following proposition that both of the agents can simultaneously achieve better performance with the SE as compared to the performance achieved by the NE.

Proposition 3 (Performance of the SE in a Two-Agent Network): In a two-agent network, if there exists a cooperating NE $\mathbf{x}'$, then there exists an SE $\mathbf{x}^*$ that satisfies

$$f_k(\mathbf{x}^*) \leq f_k(\mathbf{x}')$$

for both agents $k \in \{1, 2\}$.

Proof: Please refer to Appendix B for the proof. □

Proposition 3 shows that the SE decreases the cost functions of both agents as compared with the NE solution. Hence the SE achieves better performance for both agents in two-agent networks.

Remark 1 (Asymmetric Solution in the Stackelberg Game): In the Stackelberg game, the leader agent (internal node) picks its power action x_{12} not based on the tentative power allocation x_{21}, but the entire best-response function $T_2(x_{12})$ of the follower agent. Hence, the leader can act more intelligently, which benefits both agents. Note that there would be a different SE by choosing a different agent as the leader in a two-agent network due to the asymmetric information structure.

Remark 2 (Coordination for Leader-Follower Determination): We assume the agents know the local topology after link selection, i.e., the number of neighbors $|\mathcal{N}(j)|$ of an agent's neighbor $j \in \mathcal{N}(k)$. As a result, for a network with more than two agents, the leader can be identified as $|\mathcal{N}(k)| > 1$, and the follower can be identified as $|\mathcal{N}(k)| = 1$. For two-agent networks, the leader can be randomly assigned as the coordination overhead should be negligible in such a small network.

D. Existence of SE

It is important to study the existence of the SE, because if the SE does not exist, leader agents may waste coordination overhead for power iterations but reach zero power allocation in the end.

We first study the existence of the SE in a star network as a simple case of a tree which consists of at most one internal node. In a star network, the existence of the SE corresponds to the condition on the non-zero solution to the power allocation problem (19). The result is summarized as follows.

Definition 5 (Weakly Cooperating Link): The link (k, j) that connects a leader agent $k \in \mathcal{K}_{\text{int}}$ and a follower agent $j \in \mathcal{N}_{\text{le}}(k)$ is called a weakly cooperating link if the channel quality ξ_{kj} satisfies

$$\xi_{kj} > \left(\sqrt{\Gamma_{jk}} + \sqrt{\Gamma_{jk} + 4\Gamma_{kj}} \right)^2. \tag{22}$$

Theorem 1 (Existence and Uniqueness of SE in a Star Network): Given an agent partition $\{\mathcal{K}_{\text{int}}, \mathcal{K} \backslash \mathcal{K}_{\text{int}}\}$ and a star network topology, game $\mathcal{G}$ admits a unique SE if and only if the network has at least one weakly cooperating link.

Proof: Please refer to Appendix C for the proof. □

In addition, the following result can be established for the existence of an SE in a general tree network topology.

Theorem 2 (Existence of SE in a Tree): Given an agent partition $\{\mathcal{K}_{\text{int}}, \mathcal{K}\backslash\mathcal{K}_{\text{int}}\}$ and a tree network topology, the SE exists if one of the following is satisfied: (i) the tree has at least one weakly cooperating link between an internal node and a leaf node, or (ii) the tree has at least one link between internal nodes that satisfies (15).

Proof: Please refer to Appendix D for the proof. □

Finally, the following corollary summarizes the property of the proposed link selection and power allocation in this section.

Corollary 1: The link selection and power allocation profile $\mathbf{x}^*$ obtained from Algorithm 1 under $\bar{L} = 1$ followed by the solution of the Stackelberg game (19)–(20), is an SE if the conditions in Theorem 2 are satisfied.

IV. Game Under General Link Constraint

We study the power allocation under the general case of $\bar{L} > 1$ in game $\mathcal{G}$. From the link selection in Algorithm 1, each agent has least $\bar{L}$ links, and thus the power control vector $\mathbf{x}_k$ has at least $\bar{L}$ entries. If the NE is used as the solution concept, each agent needs to perform the best-response for power allocation by minimizing $f_k(\mathbf{x}_k, \mathbf{x}_{-k})$ given $\mathbf{x}_{-k}$ and many iterations are needed. However, this is not feasible in a large network, because of the high computational complexity and significant coordination overhead. Specifically, each agent in each turn needs to solve an optimization problem with variable $\mathbf{x}_k$ in high dimension and no closed-form solution is known. In addition, agent k needs to broadcast a vector $\mathbf{x}_k$ to the network, and hence the best-response iteration in the network may take a long time to converge. Moreover, the SE concept cannot be used because a hierarchical information structure may not exist for $\bar{L} > 1$.

In this section, we aim to find an efficient power allocation strategy for $\bar{L} > 1$, following a per link negotiation mechanism, where each agent allocates power on each link by solving a link's individual optimization problem. The power constraints of each agent is then handled by the Lagrangian multiplier (a scalar). The advantage of such per link negotiation is that first, there exists a closed-form solution to the link's individual optimization, and second, each agent only needs to iterate a scalar, rather than a vector, with the other agents.

With the per link negotiation mechanism, a new solution concept called the link bargaining equilibrium (LBE) is introduced. Consider a link's individual cost function (12); the LBE is defined as follows.

Definition 6 (Link Bargaining Equilibrium (LBE)): A solution profile $\mathbf{x}^*$ is called the LBE of game $\mathcal{G}$ if for each agent k

$$f_{kj}(x_{kj}^*, \mathbf{x}_{-k}^*) \le f_{kj}(x_{kj}, \mathbf{x}_{-k}^*), \quad \forall j \ne k \tag{23}$$

for all $\mathbf{x}_k = (\{x_{kj}\}_{j\in\mathcal{N}(k)}) \in \mathcal{P}_k$.

Note that in two-agent networks, the LBE coincides with the NE since there is only one link.

A. Power Allocation via Per Link Bargaining

To decompose the problem into per link individual optimizations, we consider the Lagrangian method as follows.

1) Lagrangian Reformulation: For each agent k, to minimize the cost function $f_k(\mathbf{x}_k, \mathbf{x}_{-k})$ over $\mathbf{x}_k \in \mathcal{P}_k$, a Lagrangian decomposition method can be used. From (5) and (6), the Lagrangian function for agent k is given as follows:

$$m_k(\mathbf{x}_k, \mathbf{x}_{-k}, \lambda_k) = \text{tr}\{\mathbf{J}_k(\mathbf{x}_k, \mathbf{x}_{-k})^{-1}\} + V_k \sum_{j\in\mathcal{N}(k)} x_{kj} + \lambda_k\Big(\sum_{j\in\mathcal{N}(k)} x_{kj} - P^{(k)}\Big)$$

where $\lambda_k \ge 0$ is the Lagrangian multiplier. Since the objective function $f_k(\mathbf{x}_k, \mathbf{x}_{-k})$ is convex in $\mathbf{x}_k$ and the optimization domain $\mathcal{P}_k$ is also convex, optimization theory [44] suggests that the optimal solution $\mathbf{x}_k^*(\lambda_k)$ to minimizing $f_k(\mathbf{x}_k, \mathbf{x}_{-k})$ under $\mathbf{x}_k \in \mathcal{P}_k$ can be obtained as

$$\underset{\mathbf{x}_k\ge\mathbf{0}}{\text{minimize}} \quad \text{tr}\{\mathbf{J}_k(\mathbf{x}_k, \mathbf{x}_{-k})^{-1}\} + (V_k + \lambda_k) \sum_{j\in\mathcal{N}(k)} x_{kj}$$

where there is a unique $\lambda_k^* \ge 0$ such that

$$\lambda_k^*\Big(\sum_{j\in\mathcal{N}(k)} x_{kj}^*(\lambda_k^*) - P^{(k)}\Big) = 0$$

$$\sum_{j\in\mathcal{N}(k)} x_{kj}^*(\lambda_k^*) \le P^{(k)}.$$

Let $\widetilde{V}_k = V_k + \lambda_k$. The constrained minimization problem for $f_k(\mathbf{x}_k, \mathbf{x}_{-k})$ can be written as

$$\underset{\mathbf{x}_k\ge\mathbf{0}}{\text{minimize}} \quad \widetilde{f}_k(\mathbf{x}_k, \mathbf{x}_{-k}, \widetilde{V}_k) \tag{24}$$

where

$$\widetilde{f}_k(\mathbf{x}_k, \mathbf{x}_{-k}, \widetilde{V}_k) = \text{tr}\{\mathbf{J}_k(\mathbf{x}_k, \mathbf{x}_{-k})^{-1}\} + \widetilde{V}_k \sum_{j\in\mathcal{N}(k)} x_{kj}.$$

The parameter $\widetilde{V}_k \ge V_k$ is to be optimized such that $(\widetilde{V}_k^* - V_k)(\sum_{j\in\mathcal{N}(k)} x_{kj}^*(\widetilde{V}_k) - P^{(k)}) = 0$ and $\sum_{j\in\mathcal{N}(k)} x_{kj}^*(\widetilde{V}_k^*) \le P^{(k)}$.

2) Link's Individual Cost Minimization: Given Lagrangian parameters $\widetilde{V}_k$ and $\widetilde{V}_j$ on link (k, j), the power allocations x_{kj}^* and x_{jk}^* are obtained from solving the link's individual cost minimization

$$\underset{x_{kj}\ge 0}{\text{minimize}} \ f_{kj}(x_{kj}, \mathbf{x}_{-k}; \widetilde{V}_k) \tag{25}$$

$$\underset{x_{jk}\ge 0}{\text{minimize}} \ f_{jk}(x_{jk}, \mathbf{x}_{-j}; \widetilde{V}_j) \tag{26}$$

where the notation $f_{kj}(.; \widetilde{V}_k)$ emphasizes that parameter $\widetilde{V}_k = V_k + \lambda_k$ is used instead of V_k.

In fact, there is a unique closed-form NE solution to (25) and (26).

Lemma 3 (Power Allocation in Per Link Negotiation): The solution to (25) is given as

$$x_{kj}^*(\widetilde{V}_k, \widetilde{V}_j) = \max\left\{0, \ \frac{a_{kj}a_{jk} - 1}{b_{kj}(1 + a_{jk})}\right\} \tag{27}$$

Algorithm 2 Power Allocation via Per Link Negotiation

1) For each $k \in \mathcal{K}$, initialize $\mathcal{N}(k)$ using link selection Algorithm 1. Let $\widetilde{V}_k(0) = V_k$.
2) At the nth step, each agent k acquires its neighbors' parameters $\{\widetilde{V}_j(n-1)\}_{j\in\mathcal{N}(k)}$.
3) Each agent k updates the power allocation $\mathbf{x}_k = (\{x^*_{kj}\}_{j\in\mathcal{N}(k)})$, where $x^*_{kj}(\widetilde{V}_k(n-1), \widetilde{V}_j(n-1))$ are given in (27) and $\widetilde{V}^*_k$ is computed to satisfy $(\widetilde{V}^*_k - V_k)(\sum_{j\in\mathcal{N}(k)} x^*_{kj}(\widetilde{V}^*_k, \widetilde{V}_j) - P^{(k)}) = 0$ and $\sum_{j\in\mathcal{N}(k)} x^*_{kj}(\widetilde{V}^*_k, \widetilde{V}_j) \le P^{(k)}$.
4) The updated parameters $\widetilde{V}_k(n) = \widetilde{V}^*_k$ are broadcasted to the network.
5) Repeat from Step 2) until convergence.

where $a_{kj} = 2\sqrt{\xi_{kj}/\widetilde{\Gamma}_{kj}} - 1$, $a_{jk} = 2\sqrt{\xi_{kj}/\widetilde{\Gamma}_{jk}} - 1$, $b_{kj} = 4\xi_{kj}\left(\delta_{kj} + \delta_{jk}\right)$, and $\widetilde{\Gamma}_{kj} = \widetilde{V}_k/(\mathbf{u}^{\mathrm{T}}_{kj}(\mathbf{J}^{\mathrm{o}}_k)^{-2}\mathbf{u}_{kj})$. Moreover, $x^*_{kj}(\widetilde{V}_k, \widetilde{V}_j)$ is non-increasing with $\widetilde{V}_k$.

Proof (Sketch): The closed-form solution can be obtained by solving a fixed point equation using the results in Lemma 4 (see Appendix A). Moreover, the non-increasing property can be established by verifying $\partial x^*_{kj}/\partial \widetilde{V}_k \le 0$. □

Using Lemma 3, the power allocation $\mathbf{x}_k$ for all the links of agent k can be obtained by computing the optimal parameter $\widetilde{V}^*_k$ such that $(\widetilde{V}^*_k - V_k)(\sum_{j\in\mathcal{N}(k)} x^*_{kj}(\widetilde{V}^*_k, \widetilde{V}_j) - P^{(k)}) = 0$ and $\sum_{j\in\mathcal{N}(k)} x^*_{kj}(\widetilde{V}^*_k, \widetilde{V}_j) \le P^{(k)}$. The agents then iterate the Lagrangian multipliers $\widetilde{V}_k$ with each other in the network. Since $x^*_{kj}(\widetilde{V}_k, \widetilde{V}_j)$ is non-increasing with $\widetilde{V}_k$, the optimal $\widetilde{V}^*_k$ can be easily found using bisection search.

To summarize, the power allocation via per link negotiation can be implemented using Algorithm 2. Note that Algorithm 2 requires low computational complexity at the agents and low coordination overhead between agents. First, the power allocation solution is given in closed-form (27), and $\widetilde{V}^*_k$ in Step 3) can be computed via bisection search. Second, each agent k only needs to broadcast and update a scalar $\widetilde{V}_k$.

B. Uniqueness of the LBE

The connection between the stationary point of Algorithm 2 and the LBE can be established as follows.

Proposition 4 (Link Bargaining Equilibrium): Every stationary point of Algorithm 2 is an LBE.

Proof: Please refer to Appendix E. □

We then study the uniqueness of the stationary point of Algorithm 2. Since the power allocation $\mathbf{x}$ is uniquely determined by $\{\widetilde{V}_k\}_{k\in\mathcal{K}}$ according to (27), the remaining question is whether there exists a unique set of variables $\{\widetilde{V}^*_k\}$ from Algorithm 2.

First, the uniqueness of $\{\widetilde{V}^*_k\}$ is straight forward under a high power budget scenario, where either $V_k \gg 1$ or $P^{(k)}$ is large enough. This is because in the high power budget scenario, the power constraint is not active, and hence $\widetilde{V}^*_k = V_k$, which does depend on $\widetilde{V}^*_j$; i.e., the power allocation problem is decoupled among agents. This insight is summarized as follows.

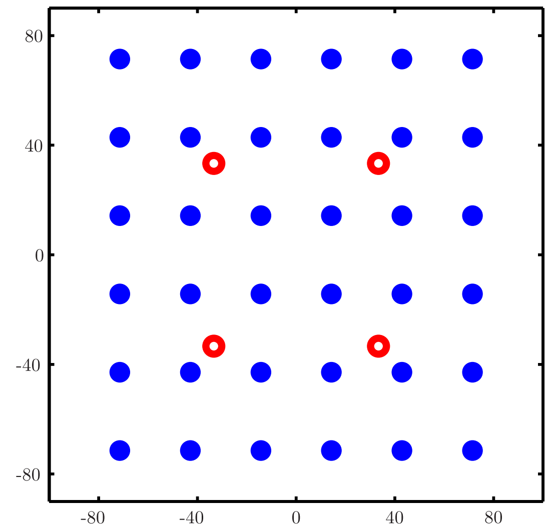

Fig. 2. Deployment of anchors (red circles) and agents (blue dots).

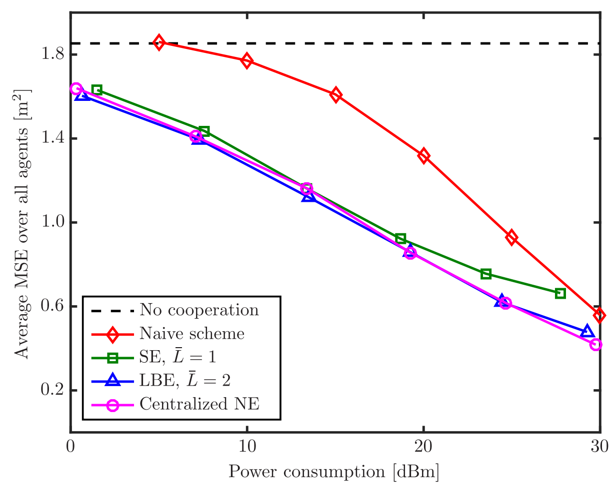

Fig. 3. Average MSE of the position estimation over all the agents.

Proposition 5 (Unique LBE Under High Power Budget): There exists $\bar{V}, \bar{P} > 0$, such that for $V_k > \bar{V}$ and $P^{(k)} > \bar{P}$, $\forall k \in \mathcal{K}$, the LBE is unique given any topology after link selection. In addition, $\widetilde{V}^*_k = V_k$ for all $k \in \mathcal{K}$.

Second, for general networks, the following result is derived for a simplified model of the initial EFIMs $\mathbf{J}^{\mathrm{o}}_k$.

Theorem 3 (Uniqueness): Suppose $\mathbf{J}^{\mathrm{o}}_k = \sigma^2_k \mathbf{I}_2$ for all $k \in \mathcal{K}$. Then there exists a unique LBE for every topology after link selection.

Proof: Please refer to Appendix F for the proof. □

As a practical implication of Theorem 3, the agents can round the EFIM into a scaled identity matrix $\mathbf{J}^{\mathrm{o}}_k \approx \sigma^2_k \mathbf{I}_2$ so as to achieve the guaranteed unique LBE under Algorithm 2.

V. Numerical Results

In this section, we evaluate the performance of the game-theoretic resource management algorithms for cooperative localization.

The network topology, where there are four anchors (red circles) and 36 agents (blue dots) all located in a 160 m × 160 m area, is depicted in Fig. 2. The ranging signals are transmitted at carrier frequency $f_c = 5.25$ GHz

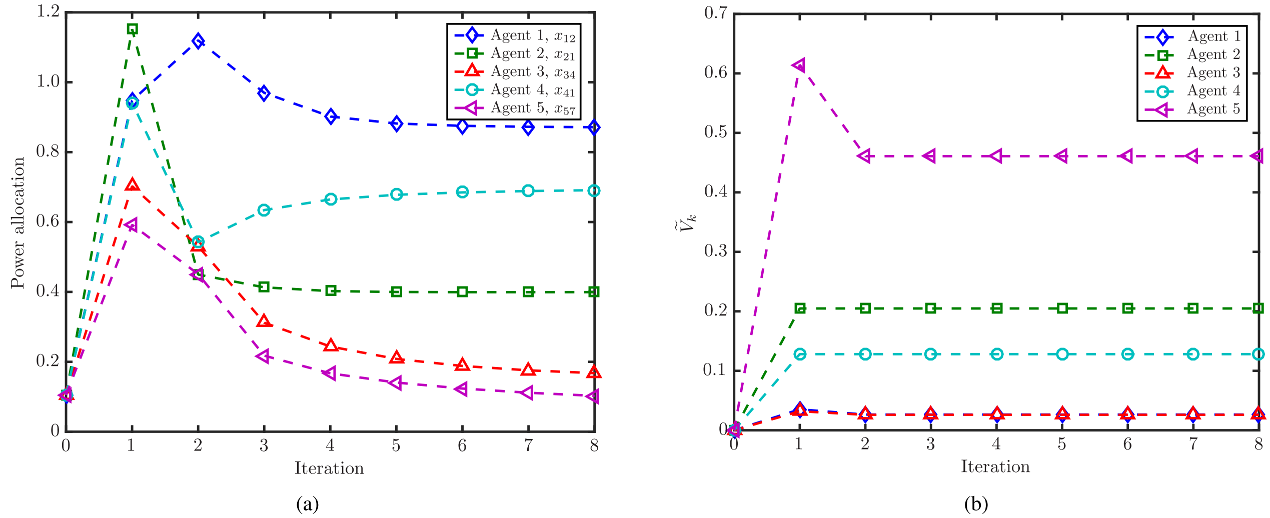

Fig. 4. Convergence for power allocation: (a) brute force best-response power iteration to find the NE under given link selection; (b) iteration to find the LBE using Algorithm 2.

with 20 MHz bandwidth. Extended WINNER channel models under the line-of-sight (LOS) cases in the indoor small office scenario and the typical urban micro-cell scenario [45] are adopted to model the propagation of the ranging signals transmitted from anchors and agents, respectively. The noise power spectral density is -168 dBm/Hz and the noise figure is 5 dB. The channel quality ξ_{kj} (i.e., equivalent ranging coefficient) is calculated according to the formulas in [5]. The agents are not synchronized with each other.

The position estimation is carried out in two phases. In the first phase, the anchors broadcast one-way ranging signals to the network with transmission power of 33 dBm. Each agent obtains an initial position estimation $\hat{\mathbf{p}}_k^{\text{o}}$, and the estimation error is roughly considered to be Gaussian distributed with zero mean and covariance matrix $(\mathbf{J}_k^{\text{o}})^{-1}$, where $\mathbf{J}_k^{\text{o}}$ is obtained according to [5, Th. 1].

In the second phase, the agents negotiate with each other for link selection and the power allocation in order to perform cooperative range measurements. Two-way TOA range measurements are taken between agents. The ranging between agent k and j is modeled as $z_{kj} = d_{kj} + w_{kj}$, where $d_{kj} = \|\mathbf{p}_k - \mathbf{p}_j\|$ is the true distance between them and $w_{kj} \sim \mathcal{N}(0, \lambda_{kj}^{-1})$ is a Gaussian random variable, in which $\lambda_{kj} = 4x_{kj}x_{jk}\xi_{kj}/(x_{kj} + x_{jk})$ [8]. A maximum a posteriori (MAP) algorithm is used for cooperative localization. In particular, the position estimate of $\mathbf{p}_k$ is given by

$$\hat{\mathbf{p}}_k = \arg\min_{\mathbf{p}_k \in \mathbb{R}^2} \sum_{j \in \mathcal{K}, \eta_{kj}=1} \frac{\lambda_{kj}}{1 + \lambda_{kj}\delta_{kj}} (z_{kj} - \|\mathbf{p}_k - \hat{\mathbf{p}}_j\|)^2 + (\mathbf{p}_k - \hat{\mathbf{p}}_k^{\text{o}})^{\text{T}} \mathbf{J}_k^{\text{o}} (\mathbf{p}_k - \hat{\mathbf{p}}_k^{\text{o}}).$$

The power budget $P^{(k)} = P_{\text{T}}$ of the agents ranges from 3 to 30 dBm. The power-conservative parameter V_k in the proposed strategies is chosen as $V_k = V/P_{\text{T}}$, where $V = 0.02$. The performance of the proposed strategies is compared to two baselines:

- Baseline 1 (Naive link selection and power allocation): Each agent selects the links with SNR above 20 dB and allocates the total power budget uniformly to the selected links.
- Baseline 2 (Centralized NE): The link selection and power allocation is obtained as the NE of game $\mathcal{G}$ computed by the best-response iteration but dropping the link selection constraints (8)–(10).

TABLE I

NUMBER OF LINKS SELECTED FOR COOPERATION

Pwr. $P^{(k)}$	SE, $\bar{L} = 1$	LBE, $\bar{L} = 2$	Naive Alg.	Centr. NE
10 dBm	19	19	470	18
20 dBm	28	50	618	60
30 dBm	31	58	628	143

Note that Baselines 2 requires global information explicitly, and hence it needs centralized computing or many iterations among agents with global information exchange. In addition, it ignores the link selection constraints.

A. Position Estimation and Link Selection

Fig. 3 shows the average MSE of the position estimation over the average power spent by the agents. In general, the proposed schemes do not use the whole power budget under poor channel quality due to the power-conservative parameters V_k. The following observations are made. First, it is shown that the proposed schemes achieve much lower MSE than the naive scheme. At low- to medium-power regimes (i.e., less than 20 dBm), the proposed schemes save more than 10 dB power to achieve a similar MSE performance. Second, in the high-power regime, although the naive scheme achieves similar MSE to the proposed scheme, it requires more than 10 times the number of range measurements, as shown in Table I, which may not be feasible in practice. Third, although the LBE solution under $\bar{L} = 1$ achieves similar MSE performance to the SE solution in the low- to medium-power regimes, it requires only half of the range measurements, as shown in Table I.

In addition, the LBE solution outperforms the SE solution in the high-power regime by requiring more range measurements. Fourth, the centralized NE baseline achieves a similar MSE to the LBE solution, although it utilizes global information explicitly and ignores the link selection constraints (8)–(10). The possible reason is that both the LBE and NE solutions are not Pareto optimal as they force the agents to act in a selfish way. The LBE solution exploits heuristics for a proper link selection, which yields a unique power allocation, whereas for the NE solution, the best-response iteration may converge to a solution that gives poor performance. In summary, with proper link selection and power allocation, as given by the proposed strategies, lower MSE can be achieved with fewer range measurements.

B. Convergence

Fig. 4 demonstrates the convergence speed of the power allocation algorithms. Fig. 4(a) shows the brute force best-response power iteration to find the NE under a given link selection, where each agent needs to update and broadcast a power vector at each iteration. Five power allocation variables are randomly selected for demonstration. Fig. 4(b) shows the iteration of the Lagrangian variable $\widetilde{V}_k$ to find the LBE using Algorithm 2. Note that, given $\widetilde{V}_k$, the vector of the power allocation $\mathbf{x}_k$ is uniquely determined. It is demonstrated that the Lagrangian variables converge after three iterations, which implies the convergence of the power allocation. These results demonstrate that the proposed algorithm converges faster and hence requires less signaling overhead.

VI. Conclusion

This paper has addressed the link selection and power allocation problems for distributed network localization, aiming to increase the resource efficiency utilized for cooperation among agents. Resource management games were formulated under constraints on the number of total range measurements taken by all the agents. A distributed link selection algorithm was derived based on a per link negotiation technique. When the link selection yields a tree topology of the agent network, the SE is used as the solution concept to exploit the hierarchical information structure for efficient power allocation. Otherwise, the LBE is used as the solution concept, which can be found by an efficient per link negotiation algorithm. It was shown that the agent network has a unique LBE under some mild conditions. Numerical results showed that the proposed strategies requires fewer iterations, less message passing, and lower computational complexity at the agents. Moreover, we demonstrated that using the proposed strategies, lower MSE of position estimation can be achieved using fewer range measurements. Future work will focus on performance analysis for the number of links selected for cooperation.

Appendix A
Proof of Proposition 2

We first state the following known result.

Lemma 4 (Best-Response in a Two-Agent Game [43]): The best-response power allocation $x_k^{\mathrm{BR}} = T_{kj}(x_{jk})$ of agent $k, j \in \{1,2\}$, $k \neq j$ in a two-agent game is given by[4]

$$T_{kj}(x_{jk}) = \left\{ \frac{\left(2\sqrt{\xi_{kj}/\Gamma_{kj}} - 1\right) x_{jk}}{1 + 4\xi_{kj}\left(\delta_{kj} + \delta_{jk}\right) x_{jk}} \right\}_0^{P^{(k)}} \tag{28}$$

where the projection $\{\cdot\}_0^P$ is defined as $\{x\}_0^P = 0$ if $x < 0$, $\{x\}_0^P = P$ if $x > P$, and $\{x\}_0^P = x$ otherwise.

We thus have the following result.

Lemma 5 (Concavity of g_{kj}): For an internal node $k \in \mathcal{K}_{\mathrm{int}}$, the function $g_{kj}(x, T_{jk}(x))$ is concave and strictly increasing in $x \in [0, +\infty)$.

Proof: From the expression of $T_{jk}(x)$ in (28), there exists a threshold $0 \leq x_{\mathrm{T}} \leq P^{(j)}$ (depending on the channel ξ_{kj} and the power budget of both agents k and j) such that $T_{jk}(x)$ is strictly concavely increasing in $x \in [0, x_{\mathrm{T}})$, and constant in $x \in [x_{\mathrm{T}}, P^{(j)}]$. Hence we have $T'_{jk}(x) > 0$ and $T''_{jk}(x) < 0$ in $x \in [0, x_{\mathrm{T}})$. Moreover, from the expression of the gain function in (3), one can show that $\partial g_{kj}(x,y)/\partial x > 0$, $\partial g_{kj}(x,y)/\partial y > 0$, and the Hessian matrix $\nabla^2 g_{kj}(x,y)$ is negative definite in $(x,y) \in \mathbb{R}_+^2$.

Therefore,

$$\begin{aligned}
&\frac{d}{dx} g_{kj}(x, T_{jk}(x)) \\
&\quad = \frac{\partial}{\partial x} g_{kj}(x, T_{jk}(x)) + \frac{\partial}{\partial y} g_{kj}(x,y)\Big|_{y=T_{jk}(x)} \frac{d}{dx} T_{jk}(x) \\
&\quad > 0
\end{aligned}$$

and

$$\begin{aligned}
&\frac{d^2}{dx^2} g_{kj}(x, T_{jk}(x)) \\
&= \frac{d}{dx}\left[\frac{\partial}{\partial x} g_{kj}(x, T_{jk}(x)) + \frac{\partial}{\partial y} g_{kj}(x,y)\Big|_{y=T_{jk}(x)} \frac{d}{dx} T_{jk}(x)\right] \\
&= \frac{\partial^2 g_{kj}}{\partial x^2} + \frac{\partial^2 g_{kj}}{\partial y \partial x}\frac{dT_{jk}}{dx} + \frac{\partial^2 g_{kj}}{\partial x \partial y}\frac{dT_{jk}}{dx} \\
&\quad + \frac{\partial^2 g_{kj}}{\partial y^2}\left(\frac{dT_{jk}}{dx}\right)^2 + \frac{\partial g_{kj}}{\partial y}\frac{d^2 T_{jk}}{dx^2} \\
&= \begin{bmatrix} 1 \\ \frac{dT_{jk}}{dx} \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} \frac{\partial^2 g_{kj}}{\partial x^2} & \frac{\partial^2 g_{kj}}{\partial x \partial y} \\ \frac{\partial^2 g_{kj}}{\partial y \partial x} & \frac{\partial^2 g_{kj}}{\partial y^2} \end{bmatrix} \begin{bmatrix} 1 \\ \frac{dT_{jk}}{dx} \end{bmatrix} + \frac{\partial g_{kj}}{\partial y}\frac{d^2 T_{jk}}{dx^2} \\
&= \begin{bmatrix} 1 \\ \frac{dT_{jk}}{dx} \end{bmatrix}^{\mathrm{T}} \nabla^2 g_{kj}(x, T_{jk}(x)) \begin{bmatrix} 1 \\ \frac{dT_{jk}}{dx} \end{bmatrix} + \frac{\partial g_{kj}}{\partial y}\frac{d^2 T_{jk}}{dx^2} \\
&< 0
\end{aligned}$$

for $x \in [0, \infty)$.

This shows that $g_{kj}(x, T_{jk}(x))$ is strictly concavely increasing in x. □

We also take note to the following fact. Given a convex function $h : \mathbb{R}^{n \times n} \mapsto \mathbb{R}$ and a linear function $g : \mathbb{R}^m \mapsto \mathbb{R}^{n \times n}$, the function $f = h \circ g : \mathbb{R}^m \mapsto \mathbb{R}$ is convex. As a result, the function $h_k(\mathbf{g}_k) = \mathrm{tr}\{[\mathbf{J}_k^{\mathrm{o}} + \sum_j g_{kj}\mathbf{u}_{kj}\mathbf{u}_{kj}^{\mathrm{T}}]^{-1}\}$ is convex in $\mathbf{g}_k$. Since the function f_k^{c} can be written as $f_k^{\mathrm{c}}(\mathbf{x}) = h_k(\mathbf{g}_k(\mathbf{x}))$, the matrix $\partial^2 f_k^{\mathrm{c}}/\partial \mathbf{g}_k \partial \mathbf{g}_k^{\mathrm{T}} = \nabla^2 h_k(\mathbf{g}_k)$ is PSD. Moreover, $\partial f_k^{\mathrm{c}}/\partial g_{ki} < 0$ for each i.

[4]Note that for the two-agent case, the notation $\mathbf{x}_{-k} = x_{jk}$ becomes a scalar.

From Lemma 5, we know both of the functions $g_{kj}(x, T_{jk}(x))$ and $g_{kj}(x, y)$ are strictly concavely increasing in x. Therefore, further due to the fact that $\partial g_{ki}/\partial x_{kj} = 0$ for $i \neq j$, we have

$$\begin{aligned}\frac{\partial^2 f_k^c}{\partial x_{kj}\partial x_{ki}} &= \frac{\partial}{\partial x_{kj}}\left(\sum_l \frac{\partial f_k^c}{\partial g_{kl}}\frac{\partial g_{kl}}{\partial x_{ki}} + V_k\right) \\ &= \frac{\partial}{\partial x_{kj}}\left(\frac{\partial f_k^c}{\partial g_{ki}}\frac{\partial g_{ki}}{\partial x_{ki}} + V_k\right) \\ &= \frac{\partial^2 f_k^c}{\partial x_{kj}\partial g_{ki}}\frac{\partial g_{ki}}{\partial x_{ki}} + \frac{\partial f_k^c}{\partial g_{ki}}\frac{\partial^2 g_{ki}}{\partial x_{kj}\partial x_{ki}} \\ &= \left(\sum_l \frac{\partial^2 f_k^c}{\partial g_{kl}\partial g_{ki}}\frac{\partial g_{kl}}{\partial x_{kj}}\right)\frac{\partial g_{ki}}{\partial x_{ki}} + \frac{\partial f_k^c}{\partial g_{ki}}\frac{\partial^2 g_{ki}}{\partial x_{kj}\partial x_{ki}} \\ &= \begin{cases} \dfrac{\partial^2 f_k^c}{\partial g_{kj}\partial g_{ki}}\dfrac{\partial g_{kj}}{\partial x_{kj}}\dfrac{\partial g_{ki}}{\partial x_{ki}} + \dfrac{\partial f_k^c}{\partial g_{ki}}\dfrac{\partial^2 g_{ki}}{\partial x_{ki}^2}, & i = j \\ \dfrac{\partial^2 f_k^c}{\partial g_{kj}\partial g_{ki}}\dfrac{\partial g_{kj}}{\partial x_{kj}}\dfrac{\partial g_{ki}}{\partial x_{ki}}, & i \neq j \end{cases}\end{aligned}$$

and

$$\nabla^2 f_k^c = \frac{\partial^2 f_k^c}{\partial \mathbf{x}_k \partial \mathbf{x}_k^T} = \mathbf{G}_k \frac{\partial^2 f_k^c}{\partial \mathbf{g}_k \partial \mathbf{g}_k^T}\mathbf{G}_k + \mathbf{H}_k$$

where

$$\mathbf{G}_k = \text{diag}\left\{\frac{\partial g_{k1}}{\partial x_{k1}}, \frac{\partial g_{k2}}{\partial x_{k2}}, \cdots, \frac{\partial g_{kK}}{\partial x_{kK}}\right\}$$

and

$$\mathbf{H}_k = \text{diag}\left\{\frac{\partial f_k^c}{\partial g_{k1}}\frac{\partial^2 g_{k1}}{\partial x_{k1}^2}, \frac{\partial f_k^c}{\partial g_{k2}}\frac{\partial^2 g_{k2}}{\partial x_{k2}^2}, \ldots, \frac{\partial f_k^c}{\partial g_{kK}}\frac{\partial^2 g_{kK}}{\partial x_{kK}^2}\right\}.$$

Note that $\mathbf{G}_k$ and $\mathbf{H}_k$ are diagonal positive definite (PD) matrices, and hence $\nabla^2 f_k^c \succeq \mathbf{0}$. Therefore, f_k^c is convex.

APPENDIX B
PROOF OF PROPOSITION 3

The objective function of agent 1 is given by

$$f_1(x_{12}, x_{21}) = \text{tr}\left\{\left[\mathbf{J}_1^o + g_{12}(x_{12}, x_{21})\mathbf{D}_{12}\right]^{-1}\right\} + V_1 x_{12}$$

where $\mathbf{D}_{12} = \mathbf{u}_{12}\mathbf{u}_{21}^T$. From the optimality condition for $\mathbf{x}' = (x'_{12}, x'_{21})$ under the best-response strategy, we have $\partial f_1(x'_{12}, x'_{21})/\partial x_{12} \leq 0$ (equality achieved when the projection is active at $\mathbf{x}'$).

In a two-agent network with agent node 1 designated as the leader, the power allocation for both agents is performed as follows:

$$\underset{x_{12}\in\mathcal{P}_1}{\text{minimize}}\ f_1^c(x_{12}), \qquad \underset{x_{21}\in\mathcal{P}_2}{\text{minimize}}\ f_2(x_{21}, x_{12}).$$

Then the function f_1^c can be written as

$$f_1^c(x_{12}) = f_1(x_{12}, x_{21})\big|_{x_{21}=T_{21}(x_{12})}$$

where $T_{21}(x_{12})$ is the best response of agent 2 as given in (28). Hence under the cooperating NE $\mathbf{x}' = (x'_{12}, x'_{21})$, $f_1^c(x'_{12}) = f_1(x'_{12}, x'_{21})$ and

$$\begin{aligned}\frac{\partial f_1^c(x'_{12})}{\partial x_{12}} &= \frac{\partial f_1(x'_{12}, x'_{21})}{\partial x_{12}} + \frac{\partial f_1(x'_{12}, x'_{21})}{\partial x_{21}}\bigg|_{T_{21}(x_{12})}\frac{dT_{21}(x'_{12})}{dx_{12}} \\ &\leq \frac{\partial f_1(x'_{12}, x'_{21})}{\partial x_{21}}\bigg|_{T_{21}(x_{12})}\frac{dT_{21}(x'_{12})}{dx_{12}} \\ &< 0\end{aligned}$$

because $\partial f_1(x_{12}, x_{21})/\partial x_{21} < 0$, for all $x_{12}, x_{21} > 0$, and $dT_{21}(x_{12})/dx_{12} > 0$, for all $x_{12} > 0$ and $T_{21}(x_{12}) < P^{(1)}$. This means that x'_{12} is not the minimizer of $f_1^c(x_{12})$, and moreover, the minimizer x^*_{12} must satisfy

$$f_1(x^*_{12}, x^*_{21}) = f_1^c(x^*_{12}) \leq f_1^c(x'_{12}) = f_1(x'_{12}, x'_{21}). \tag{29}$$

Furthermore, the convexity of $f_1^c(x_{12})$ implies that

$$f_1^c(x'_{12}) + \frac{\partial f_1^c(x'_{12})}{\partial x_{12}}(x^*_{12} - x'_{12}) \leq f_1^c(x^*_{12}) \leq f_1^c(x'_{12})$$

which gives $\partial f_1^c(x'_{12})/\partial x_{12} \cdot (x^*_{12} - x'_{12}) \leq 0$, and hence $x^*_{12} \geq x'_{12}$.

Note that $x^*_{21} = T_{21}(x^*_{12}) = \arg\min_{x_{21}\in\mathcal{P}_2} f_2(x_{21}, x^*_{12})$. Since $\partial f_2(x_{21}, x_{12})/\partial x_{12} < 0$ for all $x_{12}, x_{21} > 0$, we must have

$$f_2(x^*_{21}, x^*_{12}) \leq f_2(x'_{21}, x^*_{12}) \leq f_2(x'_{21}, x'_{12}). \tag{30}$$

APPENDIX C
PROOF OF THEOREM 1

We first find the condition for a non-zero solution to the internal node $k \in \mathcal{K}_{\text{int}}$. Using the best-response solution $T_{jk}(x)$ in (28) and the expression $g_{kj}(x_{kj}, x_{jk})$ in (3), the derivative of the partially coordinated objective function f_k^c is given by

$$\begin{aligned}&\frac{\partial f_k^c}{\partial x_{kj}}\bigg|_{\mathbf{x}_k=\mathbf{0}} \\ &= -\text{tr}\left\{\left[\mathbf{J}_k^o + \sum_{l\neq k} g_{kl}(x_{kl}, x_{lk})\mathbf{D}_{kl}\right]^{-2}\mathbf{D}_{kj}\right\} \\ &\quad \times g'_{kj}(x_{kj}, T_{jk}(x_{kj}))\bigg|_{\mathbf{x}_k=\mathbf{0}} + V_k \\ &= -\text{tr}\left\{(\mathbf{J}_k^o)^{-2}\mathbf{D}_{kj}\right\}g'_{kj}(0, T_{jk}(0)) + V \\ &= -\mathbf{u}_{kj}^T(\mathbf{J}_k^o)^{-2}\mathbf{u}_{kj}\frac{4(2\sqrt{\xi_{kj}/\Gamma_{jk}} - 1)\xi_{kj}}{2\sqrt{\xi_{kj}/\Gamma_{jk}}} + V_k \\ &= -\mathbf{u}_{kj}^T(\mathbf{J}_k^o)^{-2}\mathbf{u}_{kj}\left[2\sqrt{\xi_{kj}}\left(2\sqrt{\xi_{kj}} - \sqrt{\Gamma_{jk}}\right) - \Gamma_{kj}\right]\end{aligned} \tag{31}$$

where $\mathbf{D}_{kj} \triangleq \mathbf{u}_{kj}\mathbf{u}_{kj}^T$.

Since the internal node optimization problem is convex, according to the minimum principle [44], the trivial solution $\mathbf{x}_k^* = \mathbf{0}$ is the optimal solution if and only if

$$(\mathbf{x}_k - \mathbf{x}_k^*)^T \frac{\partial f_k^c(\mathbf{x}_k^*; \mathbf{x}_{-k})}{\partial \mathbf{x}_k}\bigg|_{\mathbf{x}_k^*=\mathbf{0}} \geq 0, \qquad \forall \mathbf{x}_k \in \mathcal{P}_k.$$

As the feasible set $\mathcal{P}_k$ requires $x_{kj} \geq 0$ for all $j \in \mathcal{N}_{\text{int}}(k)$, $\mathbf{x}_k^* = \mathbf{0}$ will not be the optimal solution if $\partial f_k^c/\partial x_{kj}\big|_{\mathbf{x}_k=\mathbf{0}} < 0$ for some $j \in \mathcal{N}_{\text{int}}(k)$.

Using (31) and solving the inequality $\partial f_k^c/\partial x_{kj}\big|_{\mathbf{x}_k=\mathbf{0}} < 0$ for ξ_{kj}, we obtain the condition as in (22), which is the sufficient and necessary condition for a non-zero solution of the internal node k.

Since the problem is convex and there is only one internal node, the associated solution is unique and stable.

Appendix D
Proof of Theorem 2

The result can be proven by contradiction. Suppose the SE does not exist. Consider that condition (i) in Theorem 2 is satisfied. Then Theorem 1 guarantees the existence of an SE in the star subnetwork. Consider that condition (ii) is satisfied. Then the SE problem degenerates to the NE problem between the two internal nodes when they both allocate zero power to their leaf nodes. Then Lemma 2 guarantees the existence of non-zero power allocation between the two nodes. By contradiction, the result is proven.

Appendix E
Proof of Proposition 4

Suppose the stationary point of Algorithm 2 is specified by $\{\widetilde{V}_k^*\}_{k\in\mathcal{K}}$, and the power allocation is given by $x_{kj} = x_{kj}^*(\widetilde{V}_k^*, \widetilde{V}_j^*)$, for all $j \in \mathcal{N}(k)$ according to (27). It is sufficient to focus on one agent, and there are three cases.

First, consider a power allocation that violates the link selection in Step 1) of Algorithm 2, i.e., $x_{kj} > 0$ for some $j \in \mathcal{K}\backslash\mathcal{N}(k)$. Since (k, j) is not selected, we must have $x_{jk} = 0$. Then $f_{kj}(x_{kj}, \mathbf{x}_{-k}) > f_{kj}(0, \mathbf{x}_{-k})$ since the gain function (3) $g_{kj}(x_{kj}, x_{jk}) = 0$, but the penalty term $V_k x_{kj} > 0$ in (12). As a result, we only need to verify the power allocation x_{kj} for $j \in \mathcal{N}(k)$.

Second, suppose for agent k that $\widetilde{V}_k^* = V_k$. Then $x_{kj}^*(V_k, \widetilde{V}_j^*)$ is the best-response solution to minimize (12), and hence condition (23) is satisfied for agent k.

Third, suppose for agent k that $\widetilde{V}_k^* > V_k$. Then it means $\sum_{j\in\mathcal{N}(k)} x_{kj}^*(\widetilde{V}_k^*, \widetilde{V}_j^*) = P^{(k)}$ according to the optimization theory for the Lagrangian method. From (27), we have $x_{kj}^*(\widetilde{V}_k^*, \widetilde{V}_j^*) \le x_{kj}^*(V_k, \widetilde{V}_j^*)$, which means the solution to minimize the individual link objective function (12) is greater than x_{kj}^*. Note that $f_{kj}(x_{kj}, \mathbf{x}_{-k})$ is convex in x_{kj}, and hence we can find $x'_{kj} > x_{kj}^*$ such that $f_{kj}(x'_{kj}, \mathbf{x}^*_{-k}) < f_{kj}(x^*_{kj}, \mathbf{x}^*_{-k})$. However, since in order to have $f_{kj}(x'_{kj}, \mathbf{x}^*_{-k}) \le f_{kj}(x^*_{kj}, \mathbf{x}^*_{-k})$ for all $j \in \mathcal{N}(k)$, we must have $x'_{kj} \ge x^*_{kj}$ for all $j \in \mathcal{N}(k)$, which violates the sum power constraint $\sum_{j\in\mathcal{N}(k)} x'_{kj} > \sum_{j\in\mathcal{N}(k)} x^*_{kj}(\widetilde{V}_k^*, \widetilde{V}_j^*) = P^{(k)}$. By contradiction, condition (23) is satisfied for agent k.

Since the above arguments apply to all agents, according to (23), the stationary point of Algorithm 2 is an LBE.

Appendix F
Proof of Theorem 3

Step 1: We examine the condition of the NE in the per link negotiation game.

Consider the power allocation $x_{kj} = x^*_{kj}(\widetilde{V}_k, \widetilde{V}_j)$ and $x_{jk} = x^*_{jk}(\widetilde{V}_j, \widetilde{V}_k)$ on link (k, j). From Lemma 3, they are the unique solutions to the following fixed point equations

$$x_{kj} = \frac{(2\sqrt{\xi_{kj}/\widetilde{\Gamma}_{kj}} - 1)x_{jk}}{1 + 4\xi_{kj}(\delta_{kj} + \delta_{jk})x_{jk}}$$

$$x_{jk} = \frac{(2\sqrt{\xi_{kj}/\widetilde{\Gamma}_{jk}} - 1)x_{kj}}{1 + 4\xi_{kj}(\delta_{kj} + \delta_{jk})x_{kj}}$$

which can be written as

$$\frac{4\xi_{kj}x_{jk}^2}{(x_{kj} + x_{jk} + 4\xi_{kj}(\delta_{kj} + \delta_{jk})x_{kj}x_{jk})^2} - \widetilde{\Gamma}_{kj} = 0 \tag{32}$$

$$\frac{4\xi_{kj}x_{kj}^2}{(x_{kj} + x_{jk} + 4\xi_{kj}(\delta_{kj} + \delta_{jk})x_{kj}x_{jk})^2} - \widetilde{\Gamma}_{jk} = 0 \tag{33}$$

where $\widetilde{\Gamma}_{kj} = \widetilde{V}_k/c_{kj}$ with $c_{kj} \triangleq \mathbf{u}_{kj}^{\mathrm{T}}(\mathbf{J}_k^{\mathrm{o}})^{-2}\mathbf{u}_{kj}$.

In particular, the parameter $\widetilde{V}_k = V_k + \lambda_k$ is chosen such that $\lambda_k = 0$ if $\sum_{j\in\mathcal{N}(k)} x_{kj}(\widetilde{V}_k) < P^{(k)}$, and otherwise, $\lambda_k > 0$ to satisfy $\sum_{j\in\mathcal{N}(k)} x_{kj}(\widetilde{V}_k) = P^{(k)}$. Equivalently, such a condition can be written as

$$\lambda_k \ge 0, \qquad \sum_{j\in\mathcal{N}(k)} x_{kj}(\widetilde{V}_k) \le P^{(k)} \tag{34}$$

$$\lambda_k\big(\textstyle\sum_{j\in\mathcal{N}(k)} x_{kj}(\widetilde{V}_k) - P^{(k)}\big) = 0. \tag{35}$$

Therefore, the NE in game $\mathcal{G}2$ is given by a set of variables $(\mathbf{x}^*, \mathbf{v}^*)$ that satisfy (32)–(35) for all links $(k, j) \in \mathcal{L}$ and all agents $k \in \mathcal{K}$, where $\mathcal{L}$ denotes the set of links in the network.

Step 2: We examine the optimality condition of a virtual global optimization problem.

Construct the following function,

$$\phi_{kj}(x_{kj}, x_{jk}) \triangleq \frac{4\xi_{kj}x_{kj}x_{jk}}{x_{kj} + x_{jk} + 4\xi_{kj}(\delta_{kj} + \delta_{jk})x_{kj}x_{jk}}$$

and consider the following constrained optimization problem,

$$\begin{aligned} \underset{\{\mathbf{x}_k \succeq \mathbf{0}\}}{\text{maximize}} \quad & \sum_{(k,j)\in\mathcal{L}} \phi_{kj}(x_{kj}, x_{jk}) - \sum_{k\in\mathcal{K}} V_k \sum_{j\in\mathcal{N}(k)} \frac{x_{kj}}{c_{kj}} \\ \text{subject to} \quad & \sum_{j\in\mathcal{N}(k)} \frac{x_{kj}}{c_{kj}} \le \sigma_k^4 P^{(k)}, \quad \forall k \in \mathcal{K}. \end{aligned} \tag{36}$$

The Lagrangian function of problem (36) is given by

$$\begin{aligned} \Phi(\mathbf{x}, \boldsymbol{\lambda}) &= \sum_{(k,j)\in\mathcal{L}} \phi_{kj}(x_{kj}, x_{jk}) \\ &\quad - \sum_{k\in\mathcal{K}} \sum_{j\in\mathcal{N}(k)} (V_k + \mu_k)\frac{x_{kj}}{c_{kj}} - \sum_{k\in\mathcal{K}} \mu_k\sigma_k^4 P^{(k)} \\ &= \sum_{(k,j)\in\mathcal{L}} \Big[\phi_{kj}(x_{kj}, x_{jk}) - \hat{\Gamma}_{kj}x_{kj} - \hat{\Gamma}_{jk}x_{jk}\Big] - \sum_{k\in\mathcal{K}} \mu_k\sigma_k^4 P^{(k)} \end{aligned}$$

where $\mu_k \ge 0$ are Lagrangian multipliers for the constraints in (36) and $\hat{\Gamma}_{kj} \triangleq (V_k + \mu_k)/c_{kj}$. Note that we have

$$c_{kj} = \mathbf{u}_{kj}^{\mathrm{T}}(\mathbf{J}_k^{\mathrm{o}})^{-2}\mathbf{u}_{kj} = \mathbf{u}_{kj}^{\mathrm{T}}(\sigma_k^2\mathbf{I}_2)^{-2}\mathbf{u}_{kj} = \sigma_k^{-4}.$$

Thus, the Karush-Kuhn-Tucker (KKT) condition to the optimization problem (36) can be simplified as

$$\frac{4\xi_{kj}x_{jk}^2}{(x_{kj} + x_{jk} + 4\xi_{kj}(\delta_{kj} + \delta_{jk})x_{kj}x_{jk})^2} - \hat{\Gamma}_{kj} = 0 \tag{37}$$

$$\mu_k \ge 0, \qquad \textstyle\sum_{j\in\mathcal{N}(k)} x_{kj} \le P^{(k)} \tag{38}$$

$$\mu_k\sigma_k^4\big(\textstyle\sum_{j\in\mathcal{N}(k)} x_{kj} - P^{(k)}\big) = 0 \tag{39}$$

for all $k \in \mathcal{K}$.

Step 3: Equivalence.

Observe that problem (36) is convex, and hence optimization theory [44] suggests that there is a unique solution

$(\mathbf{x}^*, \boldsymbol{\mu}^*)$ to satisfy conditions (37)–(39). We also observe that conditions (32)–(35) are exactly the same as the KKT conditions (37)–(39). As a result, there is a unique $(\mathbf{x}^*, \boldsymbol{\lambda}^*)$ to satisfy (32)–(35), which implies that $(\mathbf{x}^*, \mathbf{v}^*)$ is unique. This confirms that the LBE is unique.

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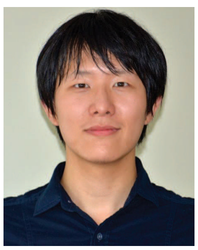

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